UNIVERSITY OF TWENTE.

Faculty of Electrical Engineering, Mathematics and Computer Science

Module 2
Electric Circuits

Laboratory Manual

Electrical Engineering
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Contents

Chapter 1

Introduction

1.1 Description of the lab exercises

Purpose and contents of the lab exercises

The lab exercises in module 2 hold a dual purpose. Some of the exercises emphasize on the responsible use of measurement instruments (see subsection Measurement Instruments). The other exercises focus on the experimental testing of the theory of the module Electric Circuits. The lab exercises end with a series of tests, in which the aspects of the experiment, the analysis of a circuit and the focus on the measurement instruments, are all integrated. The lab exercises are preceded by those of module 1. Measurement instruments like the multimeter and the oscilloscope have already been used in this module. These instruments are assumed to be known and will not be introduced again, but they will be used of course.

Place of the lab exercises in the curriculum: the line of lab exercises

During the lab exercises, the experimental skill will be learned, which is required by an important activity of an electrical engineer: the design of systems of electric or electronic nature. View the General Manual for Lab exercises (GLM) for an extended description of the first years lab exercises. The design of circuits itself is limited during the lab exercises. The focus will be on the responsible use of the measurement instruments and the testing of properties of electrical circuits by performing measurements.

The module Electric Circuits

During the module Electric Circuits a quantity of theory will be treated. The lab exercises are partly used to confront students with this theory in another way. Especially the practical testing of the theory will lead to a deeper and enlivening understanding of the acquired theoretical knowledge.

Measurement Instruments

Measurements on electrical circuits are never perfect. Roughly, there are three kinds of problems:

1. The experimenter makes a mistake; this can only be avoided by working very carefully, checking all connections in the setup thoroughly and subjecting all measuring results to a careful examination.
2. The measurement instrument affects the circuit behaviour, particularly by its “internal” resistance, which causes the currents and voltages to be different from those in the circuit without the instrument.
3. The measurement instrument itself is faulty, allowing only a limited accuracy in results.

During the lab exercises, the student will learn to use electrical measurement instruments correctly. Some of the used instruments are the multimeter (for measuring voltages and currents) and the oscilloscope. The student will learn the limitations of the instrument and will learn to be able to compensate for their influence on the circuits. Attention will be paid to methods of measurement that avoid some of the errors and make measurements with high precision possible (e.g. measuring bridges). Lastly, the student will learn to form an impression of the remaining error.

Electric Circuits

Knowledge from the lectures Electric Circuits will be put into practice in the lab. The lab also plays its own part: teaching the students to test a model in practice and learn its limitations. In addition, it is the first series of lab exercises in which skills, gained during the part Measurement Instruments, will be applied.
The topics treated will, as much as possible, appear in the same order as they do during the lectures:

- direct current (DC) circuits;
- time variant signals, alternating current (AC) circuits;
- characteristics of circuits in the time domain;
- characteristics of circuits in the frequency domain.

1.2 Organisation

Timetabling

The lab exercises consist of 7 individual exercises. Two intervals are available for each exercise (2 times 4 hours). **It is strongly recommended spending some time at home reading and preparing (!)**, as this will speed up the start during the actual lab day. View paragraph 1.3 for the exact procedure. The layout and timings can be found on the timetable. The exercises will be done in groups of two students.

The 7 individual exercises are chronologically divided as follows, in week 8 there is a possibility for a resit:

Week	Intervals (We-Thu)	Exercise/experiment
1	1-2	The diode
2	3-4	The Wheatstone bridge
3	5-6	Response of 1st and 2nd order circuits
4	7-8	The oscilloscope and the probe
5	9-10	Transfer functions and Bode diagrams of RC and RLC networks
6	11-12	Fourier analysis of signals
7	13-14	The black-box model
8	15-16	**Resit**

Manuals

In the first year there is, besides this manual, the additional General Manual for Lab exercises (GLM). The GLM can be found on the Blackboard website. The GLM contains a number of things that are also important for other lab exercises. This manual contains many references to the GLM. The theory used in the exercises is almost completely from the book by Nilsson and Riedel (hereafter denoted by “Nilsson”), used in the Electric Circuits lectures. The only exception are the bridge circuits in chapter 4.

The journal

Each group of three students will make one journal for each exercise. This journal has to be **electronically** put on the blackboard at the end of the second mid-day or interval. The journal will be graded once a week and given back the next time. The journals will be made in a notebook designed for this purpose, the laboratory journal.

Division of labour during the exercise

Because each group of three students will write one journal, the division of labour could be something like: two students “operate”, the other journalises. Easy enough, but these tasks need to be alternated. This should be done using the following scheme:

Exercise	Journalising
1	Student 1
2	Student 2
3	Student 3
etc.	

Please write in each journal which student has journalised. Mind you, the student journalising will also participate in the experiments. It's not forbidden for him/her to touch the instruments, but this student carries the main responsibility for the journal.

Rating

Every student will receive a rating of each exercise, mostly based on the journal that has been handed over. All items in the journal will be discussed: answers on star questions, validity of the circuit design, precision during the execution and the journalising of the measurements and the critically drawing of conclusions.
Normally all three students receive the same rating, but exceptions can be made when the situation regarding the exercise gives a reason for it.
It is nice when an experiment is successful, but this is not essential. Much more important is a good analysis of the way by which the results were obtained.
The assistant will inspect your commitment and working attitude. Apart from this and apart from obvious misbehaviour, you will not be judged during the practical execution in the lab. So please ask lots of questions. A complete answer is not guaranteed though: the assistant will sometimes merely give you a hint or direction to a solution.

For this lab part of the module Electric Circuits a total of 9 marks will be given, linked to the tests of the theory part of the module. The convolution part of the module will be treated in the final experiments. Experiment 3 (first and second order circuits) and experiment 4 (the oscilloscope and the probe) will result in two marks. The rest of the experiments will result in a single mark.

To pass the laboratory part of the module Electric Circuits, all experiments must be executed, the individual marks must be 5 or higher and the total average mark must be 5.5 or higher.

Impediment

When you are unexpectedly hindered from doing a lab exercise (by illness for example), you are kindly asked to contact one of the lab assistants as quickly as possible to plan a time for catching up. Please also inform your partner.

Addresses

Lab: Westzaal Zilverling, Zi A220

Lab supervisors

ing. S.M. (Sander) Smits, Carré 2615, tel. 5929
dr. ir. R.J.E. (Ray) Hueting Carré 2613, tel. 2754

Assistants

The assistants have their own specific place in the lab. This is at one of the tables in the centre.

1.3 Execution of the experiments

Layout of the lab time

All exercises will be done during the lab hours, this includes the preparation and the journalising. Two hours are scheduled for preparation, prior to the actual lab exercise. Preparation is done using the section “Analysis” of the exercise written in this manual. It consists of reading the relevant literature (this lab manual, the GLM, the book on Electric Circuits (Nilsson)). Also part of the preparation is the journalising of the sections “Preface”, “Analysis” and “Methods”. This includes answering the questions that have a double *. (e.g. question *6*)

Note: Execution of the practical assignment may only begin when the star questions mentioned above have been answered beforehand! It regularly happens that students run out of time for the preparation. It is thus advisable to read the exercise and answer the questions from the analysis the evening before the lab.

The experiment is to be conducted in accordance with the lab manual in the remaining six hours. There should be enough time for journalising. The practical assignments also contain star questions (noted with a single *, e.g. 9*). These questions need to be answered during the practical exercise. Immediately after the lab exercise, when the journalising is finished, the journal is to be handed over to the assistants for correction and grading.

Systematic approach and journalising

Practical assignments have certain requirements. The most important ones are that the results are reliable and reproducible. Reliability ensures that a thorough analysis of an experiment proves that drawn conclusions follow from the journalised measuring results and the description of the measurement setup. Reproducibility implies that all information, needed for repeating the experiment under the same circumstances, with the same results, can be found in the journal. The data need to convince the reader on the validity of the results and the conclusions.

A manual for writing a journal is found in the GLM. In this manual each lab exercise comprises systematic points that form a part of the journal. The lab manual follows these points in the practical assignments. In this way, a systematic approach and good journalising work together easily.

Layout of the journal

As mentioned above, the lab exercises are done using a number of general rules or points. Here we give a short summary of these points.

1. Introduction
   In the introduction, a problem is given and explained shortly if necessary.
2. Analysis
   The analysis addresses the theory, which is used for the necessary calculations. This can be an explanation, but most of the time the student needs to answer a number of star questions. These are the questions marked with a double * (*x*).
3. Methods
   The methods address the design of the circuit, using additional calculations and notes when necessary. Most of the time, this is also done in the form of (star) questions and assignments.
4. Execution
   Since the exercise is well prepared now, the setup can be built quickly. When this is done, measurements can be made and the journalising can be done. The actual

experiments will prove to take the least amount of time after a good preparation. We know exactly what needs to be done, after all.

5. Conclusions
A good journal always contains one or more clear conclusions, in which the results are evaluated:
- Do the results agree with the theory?
- What reasons are there for deviations?
- Are there any recommendations for possible repeating of these experiments?

Salient points will be addressed in the form of star questions with a single *.

Extensiveness of the journal

Keep the journal short and do not copy extensive stories from other sources. Rather summarize such stories (in this case please also refer to the sources) and give the results and conclusions. Write down the formulas you are going to use for your calculations, for example.
Derivations need only be given when the exercise asks for it.
As mentioned, though, the journal has to contain all data that is needed for repeating the experiment under the same circumstances. These are mainly the things that are specific for the experiment: the used equipment and the gained results.
So a journal has to be complete, but can be kept short. When you keep your journal short, it will save you and the assistant a lot of time!

The manual

The manual, especially for the first exercises, gives a lot of advices for the assignments in accordance to the layout of the journal. This will become less for the later assignments. You are expected to gradually become more independent in solving problems and you occasionally have to get your knowledge elsewhere (mostly from the lectures of Electric Circuits). You must, however, always write your journal in accordance to the format mentioned before, even if this seems less obvious in the manual. Sometimes there is a bell (🔔) indicated in the manual. This means that this is an important part of the lab exercise, so please do pay attention.

Practical hints regarding the journal

- Each group of three writes one journal.
- Write your *name and surname* on the cover. Also write down the name of your partners.
- Furthermore, write down the *name* of the lab course, “Lab module 2”, together with the year of the course and your *group’s number*.
- Write down the numbers for each *assignment* or *question.*
- Use the *right* pages of the journal only for writing. It is easier to read and the notebook has enough pages for this not to be a problem.
- **Graphs**: please put these on the *left* pages only. Use the graph paper that is distributed in the lab. Please supply each graph with a *practical scale* and make clear which variables belong to which axes.
- Use *suitable units* and complete the graph with a *caption*.
- When the exercise asks for a *scope grid*, it is not sufficient to use the oscilloscope’s settings. Derive your own scale and write it on the graph.

- When calculations have to be made, a single answer will not be enough. Write down each step of the process, just like when a derivation is asked.
- When you draw a conclusion or make a certain claim, please also write down the argumentation. Do not claim things arbitrarily.
- A flawless *spelling* is important in all forms of written communication. Also pay attention to a ***correct*** and ***professional style***.
- A *beautiful handwriting* is not required, but do write ***clearly*** and ***readable***.

Uncertainties and the error analysis

The key question of each exercise is: do the experimental results conform to the expectation derived from the theory?
This question therefore needs to be answered in every conclusion.
The theoretical expectations and the experimental results will never agree exactly, but only in a certain margin. When the margin can be explained from the specified measurement inaccuracies, we can say that the theory is confirmed by the experiments. A better expression: *the experiment does not contradict the theory*. And as long as the experiment doesn't contradict the theory, we assume that the theory is right.
Chapter 3 of the GLM contains a few techniques which can be used to analyse the accuracy of the experimental results. This is called error analysis . A few of those techniques will be practiced during the lab exercises. At the beginning of every exercise is indicated what is expected of you regarding the error analysis. This is related to the kind of experiment that will be conducted and the phenomenon that is mainly responsible for the inaccuracies in the final results.

Chapter 2

Measurements on circuits

2.1 Currents, voltages and measuring devices

The most common physical quantities in electronics are the current and potential difference (also known as voltage). There are devices to measure these quantities. For the potential difference we can use a volt meter. For measuring the current we use a moving-coil meter. An ideal measuring device doesn't influence the values of the quantities in the circuit when they are implemented.
A current can only flow in a closed circuit. If we want to measure a current, we usually look at the magnetic field they produce or a voltage drop over a so called shunt resistor. A device that measures the current is called an ammeter. An ammeter is placed in series with the component from which you want to know the current that flows through it. Positive currents are defined as a current flowing from a higher potential level (red input) to a lower potential level (black input). So if the current flows in the opposite direction (from black to red), the moving-coil meter will indicate a negative current, although some ammeters such as moving-coil meters can't indicate negative currents.
A voltage is a difference in electric potential between two points in a circuit. It can be measured either digitally or analogically. Analogue voltmeters are basically current meters with a series resistance that differs. Different series resistances allow you to measure different voltage ranges. Most voltmeters have a knob that allows you to select different voltage ranges. In more expensive voltmeters this is done automatically. Since a voltage, or potential difference, is always defined between two points in a circuit, it would be better to speak of a of a voltage or potential difference. Usually one wants to know the potential difference over a component. For this reason we connect voltmeters parallel to that component.
Voltages can also be measured with an oscilloscope. Here, an electron beam is curved by creating an electric field. This is done by applying a potential difference on two conducting plates. Just as with the ammeter, a voltage becomes negative when you connect the voltmeter the other way around.
Voltmeters are implemented just like ammeters into the circuit. Their influence on the initial values of the quantities cannot be neglected. **In both cases we will read values that are different from those of a circuit without the measuring devices.** This is something we have to take into account. We can do this by making their influence as small as possible or by correcting these changes (see chapter 2.3). The fault that is created is different from the faults you get from inaccuracy of the measurement device itself!
Voltmeters and ammeters can also be used to measure each other. By placing an ammeter in series with a resistance, parallel to a component, we can calculate a voltage by using the formula $v=i \cdot R$. Currents can be measured with voltmeters by placing a voltmeter parallel with a resistance in series with a component. The formula to calculate the current then becomes $i=u/r$.

2.2 Components and elements, network theory

Electrical systems are built up using many kinds of components. The system serves a purpose depending on the arrangement of the components. In general there are two kinds of components: active and passive components. Active components are components that are capable, optionally in combination with some sort of power supply, in increasing the power of

element	voltage-current relation	component
voltage source	$v(i,t) = e_0(t)$	voltage generator, voltage supply
current source	$i(v,t) = i_0(t)$	current generator
resistance	$v = i \cdot R$	resistor
induction	$v = L \cdot di/dt$	solenoid
capacity	$i = C \cdot dv/dt$	capacitor
transformer	$u_o/u_i = i_i/i_o$	(voltage) transformer
gyrator	$u_i = -R \cdot i_o$; $u_o = -R \cdot i_i$	electronic gyrator

a signal.
Passive components only weaken the power of a signal in some way. Some components like capacitors can temporarily store energy but never increase the total power in a signal.
By combining both passive and active components in an IC (integrated circuit), it is possible to create boxes that, on themselves, can be seen as separate active components. Operational amplifiers are a great example of such ICs.
In order to describe the behaviour of electrical systems, we create models that are built up of very fundamental components. These fundamental components are: resistors, capacitors, inductors, transformers, gyrators, voltage sources and current sources. Basically, these components summarize those relations between current and voltage, which allows us to make a model of every possible system with only these components. Table 1 summarizes the elemental equations of the components mentioned above. In the third column, you can see components that function according to these equations.

The symbols for the voltage and current source, resistor, capacitor and inductor are shown in figure 1.
In the case of an ideal voltage source, the voltage across the terminals is constant and always exactly the same. If this voltage source was short circuited, the current would be approximately infinity, which is impossible in reality.
Ideal current sources always make sure the current flowing out is exactly the same regardless of the voltage that would follow out of ohms law. This would mean that in the case of an open circuit, the voltage will be raised to infinity since $v = i \cdot r$ and r approaches infinity. Such situations are impossible thus can't occur. This besides the fact that ideal voltage and current sources don't exist anyway.
Voltages and currents don't necessarily have to be constant. They can vary in time as well, In that case we notate the signal as $v(t)$ or $i(t)$.

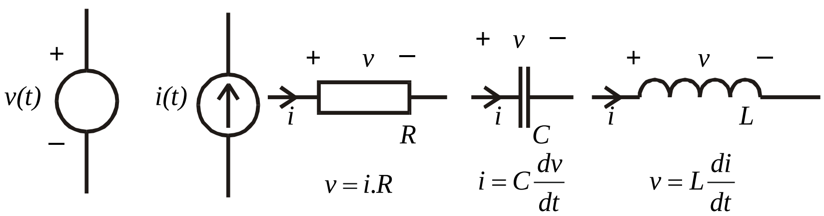


Figure 1 Symbols for a voltage source, current source, resistor, capacitor and inductor

Remember: DC = direct current implies constant voltage or current
AC = alternating current implies alternating voltage or current/time dependent voltage or current

A signal generator or function generator is a device that enables to generate alternating voltages. These signals are usually sine waves, square waves, triangle waves and saw tooth waves. These waves all have a period *T*:

$$v(t) = v(t - n \cdot T) \;\; n \in \mathbb{N} \qquad (1.)$$

2.3 The influence of power supplies and measuring devices

2.3.1 Introduction

All types of power supplies have a behaviour that is close to what we expect them to behave like. For example, we expect that a voltage source provides us with a nice constant voltage. In reality however, this is not the case. When we ask for large currents or large current changes, we will see that for most voltage supplies their voltage levels plummet. All of the other electrical components have such unwanted behaviour as well. We will see how to model these side effects in chapter 2.3.2. In the following chapters, we will see how we can take these side effects into account and contemplate whether they will affect our electric circuit.

2.3.2 Equivalent-circuit diagrams

There is a difference between circuits and networks. Networks can be sub-systems in a circuit. In networks, we find most of the network components that we addressed before. The truth is that most of these components don't behave ideally. To model these side effects we invented equivalent-circuit diagrams. These basically make a substitution for bare components that will simulate the behaviour of the components as they are in reality. We will now talk about modelling these components.

Capacitors and resistors

Even though capacitors mostly cover their function as a capacitor with capacitance C, their behaviour in real life is also afflicted with dielectric losses. It's best to model a capacitor as a capacitor with a series resistance in the form of a resistor. Just as with the capacitor, resistors also are afflicted with side effects. Just as most wire-shaped components, resistors have an induction as well. And at the same way, inductors also are afflicted with the resistance of the wound copper wire. Below, in figure 2, we can see how we can model the capacitor and the resistor.

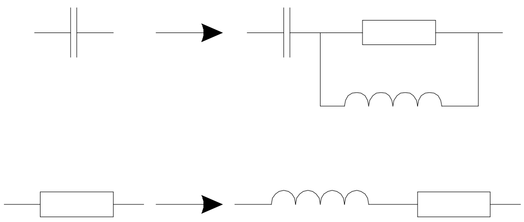


Figure 2. Possible models of the capacitor and resistor

Power supplies

We can model a voltage source by using the Thévenin rule. By adding a series resistance, which in reality is the internal resistance of the voltage supply, we can drastically increase the realism and accuracy of our voltage supply model. Below, in figure 3, we can see what such a voltage supply looks like. Please note that the series resistor is notes as R_i. The i originates from the word "internal" which describes its purpose.

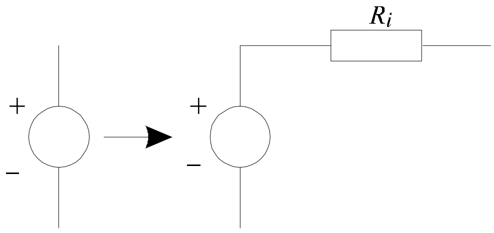


Figure 3. The modelling of a non-ideal voltage source

The current source can be modelled by using Norton. By adding an internal resistance parallel to the current source, we can accurately simulate the behaviour of a possible current source that we would find in real life. Because of this resistance, there is no such thing as an infinite voltage anymore since the internal resistance of resistor R_i is not infinitely large. Current sources however, are not very common and are usually made with more complex networks. Below in figure 4 we can see how we can model a current source.

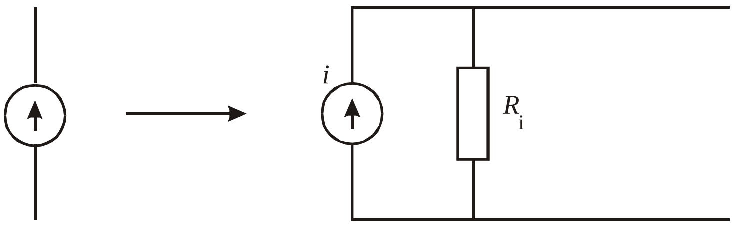


Figure 4. The modelling of a non-ideal current source

Voltmeters

Voltmeters are placed parallel to the intersection or component over which we want to know the voltage. Ideal voltage sources have an infinite resistance so their influence approaches zero. In reality however, this resistance is not infinitely large but just large. To model this, we keep our ideal model of the voltmeter and simply add an internal resistance R_i parallel to the voltmeter to model the equivalent-circuit diagram. Below, in figure 5, we can see the modelling of a voltmeter.

Please notice that because our voltmeter is ideal in our model, we can leave out this component while doing calculations because its effect is zero anyway.

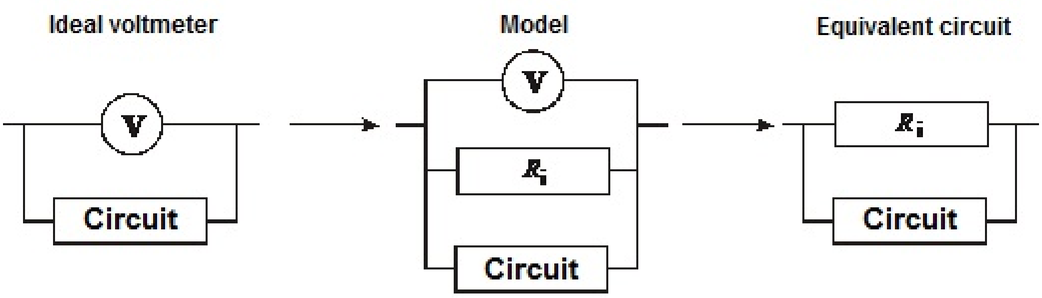


Figure 5: The modelling of a non-ideal voltmeter over a circuit

Ammeters

A current can be measured with an ammeter. This one is supposed to be placed in series with the component or circuit. In the ideal case, the internal resistance is zero but in reality, as expected, this is not the case. The ammeter has a series resistance to model the internal resistance and just as with the voltmeter, we can leave out the ammeter while working with the circuit because its effect is zero. Below, in figure 6, we find the modelling of the ammeter.

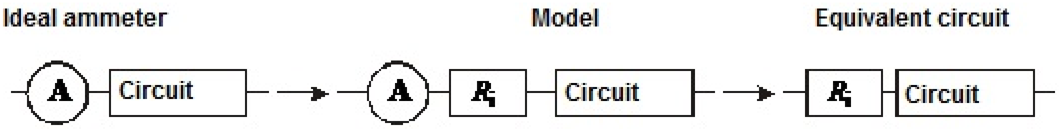


Figure 6: The modelling of the non-ideal ammeter in series with a circuit

The oscilloscope

With an oscilloscope we can measure time dependent electric signals. An oscilloscope is very similar to a voltmeter. Besides the internal parallel resistance, an oscilloscope also has an parallel capacitance C_i. The modelling can be seen in figure 7.

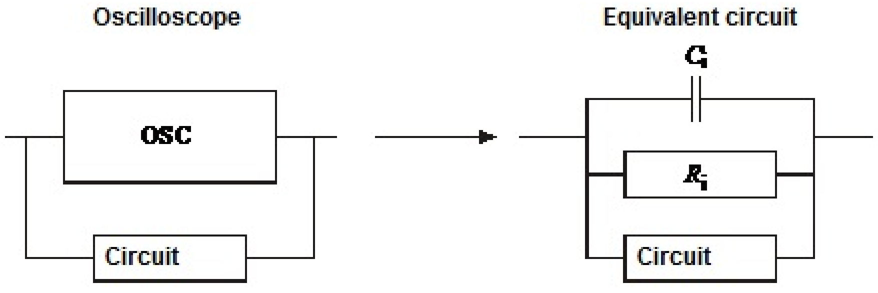


Figure 7: The modelling of a non-ideal oscilloscope measuring over a circuit.

2.3.3 Minimizing the influence of measurement devices

With the equivalent circuits above, we can take into account the use of measurement equipment in a circuit. With this information, one can take a look at the influence of the measurement equipment and whether this influence can be neglected. In 2.3.4, this influence is explicitly taken into account. As an example we take a look at two circuits for doing current-voltage measurements on a certain component *comp*.

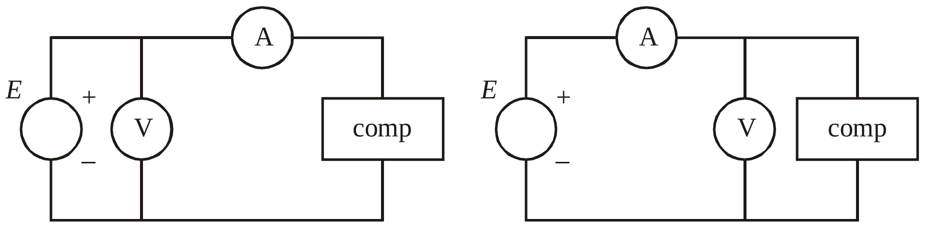


Figure 8 Circuits for current-voltage measurements.

If we can swap the measurement equipment with their equivalent circuits, we obtain the following networks.

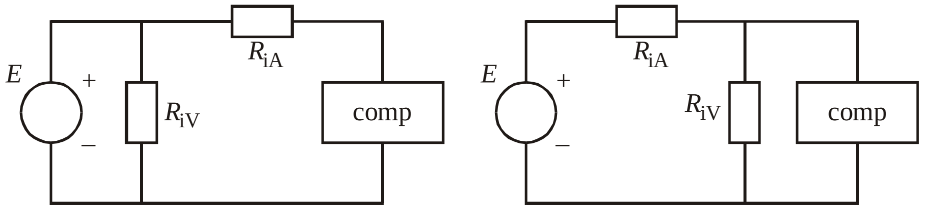


Figure 9 Equivalent circuits for the circuits depicted in figure 8

Left network

The current through the component is measured exactly. However, we do not measure the voltage over the component, but the voltage over the component in series with the ammeter:

$$v_{\text{measured}} = v_{\text{A}} + v_{\text{comp}} \qquad (2.)$$

Right network

The voltage over the component is measured exactly, however, we do not measure the current through the component, but the current through the component in parallel to the voltage meter:

$$i_{\text{measured}} = i_{\text{V}} + i_{\text{comp}} \qquad (3.)$$

Measuring resistance

Time dependent *voltages* can be measured directly using the oscilloscope. For the measurement of time varying *currents*, one can make use of a measuring resistor. In this application, one can measure the voltage over this measuring resistor with the oscilloscope. The current through the measuring resistor is proportional to the voltage with coefficient $1/R_m$. However, this resistance does have an influence on the circuit. This can be analyzed in the same way as the section above. Both channels of the oscilloscope can be considered to have a high valued impedance.

One can also use a measuring resistor to measure a direct- or alternating current which is too small to measure directly.

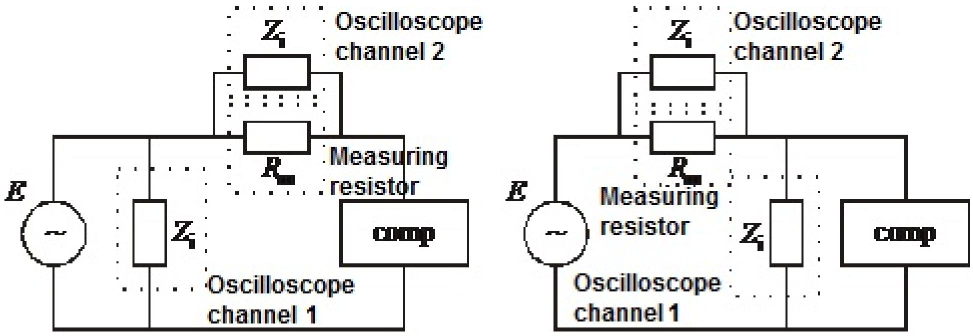


Figure 10 The use of a measuring resistor to measure current with the aid of an oscilloscope

2.3.4 Correction of voltage and current values

It is always possible to compensate for the influence of measuring equipment tools. Their properties are always specified in the user manual and/or the back panel of the instruments. As an example we will take a look at the compensation for the internal resistance of an ammeter at current-voltage measurements.

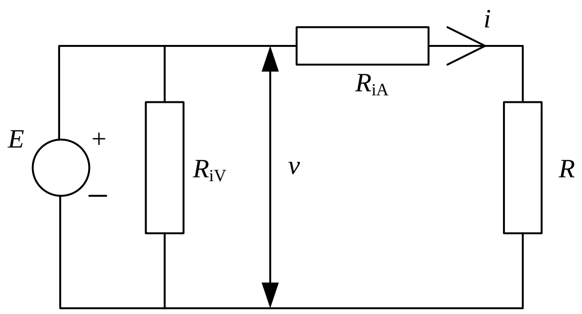


Figure 11 The internal resistance of a current- and voltage meter

Using measurements of voltage v and current I, we want to determine the resistance R. In a first indication, R is given by:

$$R = \frac{v}{i} \qquad (4.)$$

The schematic of the figure above immediately leads to a revised approximation:

$$R = \frac{v}{i} - R_{\mathrm{iA}} = \frac{v - v_{\mathrm{iA}}}{i} \quad (5.)$$

In which v_{iA} is the potential difference over the ammeter.
At least one of the two, either v_{iA} or R_{iA} is always specified for the ammeter.

At last
A complete discussion should be made regarding corrections because of the use of a measuring resistance.

2.3.5 Influences on the time dependency of signals (distortion)

The meters do not only have an influence on the amplitude of signals, but also on their shape. This, of cause, primarily counts when talking about time varying signals. Current-voltage relations can then be determined using an oscilloscope and a measuring resistor (experiments 4 and 5).

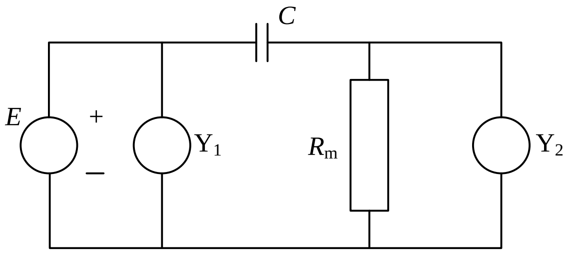


Figure 12 The measuring of current-voltage relations using an oscilloscope and a measuring resistor

In the figure above, Y_1 and Y_2 both are oscilloscope channels. To measure the current through the capacitor, *measuring resistor* with resistance R_m is placed in series with the capacitor. The current measured by Y_2 is directly proportional to the amplitude of the current i.
Suppose we switch on the voltage supply at moment t = 0 s and from that moment linearly increase the voltage E:

$$E(t) = a \cdot t \quad (6.)$$

With 'a' a certain determinable constant.

Situation without measuring instruments
Without measuring instruments, there is no measuring resistor. The voltage E is fully over the capacitor. The situation for the current is:

$$i = C\frac{dE}{dt} = a \cdot C \quad (7.)$$

Situation with measuring instruments
Now, the capacitor is in series with R_m. We now find:

$$E(t) = v_c + iR_{\mathrm{m}} \Rightarrow$$
$$\frac{i}{C} + R_{\mathrm{m}}\frac{di}{dt} = \frac{dE(t)}{dt} = a \quad (8.)$$

Effect:

$$i(t) = aC(1 - e^{-\left(\frac{t}{R_{\mathrm{m}}C}\right)}) \quad (9.)$$

Consider both situations. Also note that when the product $R_m C$ is very small, the relation in equation (9) is practically the same as the relation in equation (7).
Below, one can find plots of the amplitude of the current without a measuring resistor (the straight line, left) and the amplitude of the current in a circuit with a measuring resistor (exponential curve, right).

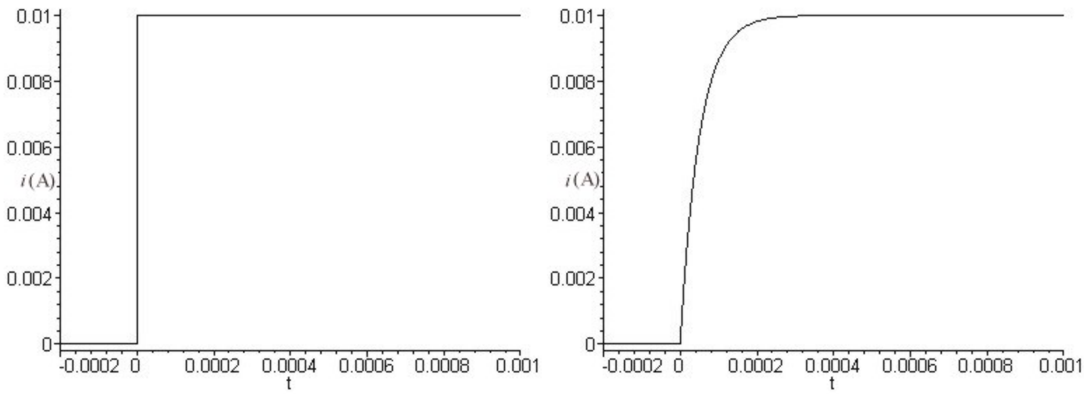


Figure 13 Left: Current without a measuring resistor. Right: current with a measuring resistor, $R = 50\ \Omega$, $C = 1\ \mu F$, $a = 1{,}0.10^4$ V/sec

We can see that, based on the ideal model, the current at moment t = 0 s instantly increases from 0 to 10 mA. The actual measured signal increases more slowly to an end value.

2.3.6 The load line of a voltage source

Suppose we have a voltage generator set to a certain voltage. Now we load the network with a certain resistance (or more general *impedance*). If we now measure the generated voltage, this value usually seems to be lower than the previously set value. The voltage "drops". We illustrate this using an example, in which the load is a resistor. Below, we can see the network in which the voltage can be replaced using the equivalent circuit.

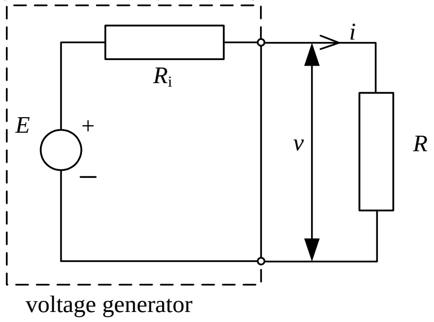


Figure 14 Equivalent circuit of a voltage generator loaded with a resistor

The ideal voltage supply provides a fixed voltage E. At the terminals of the voltage generator is voltage v, which we can measure. There is also a current i through the load resistor. The following applies:

$$\left.\begin{aligned} E &= i(R_i + R) \\ v &= iR \end{aligned}\right\} \Rightarrow v = E - iR_i \qquad (10.)$$

If we now plot in a graph the acquired voltage v against the current I, we get the *load line* of a voltage generator.
The voltage usually decreases when the current increases, so the load line will be from the upper left to the lower right in the graph. This might be counterintuitive.

2.4 Voltage divider

A common circuit is the voltage divider. The accompanying network is given below. On the left we can see two resistors in series, in which v_{in} is the voltage over both resistors and v_{out} is the resistance over just the resistor R_1. On the right we see the exact same network, only ordered differently. Now the names v_{in} and v_{out} become more clear.

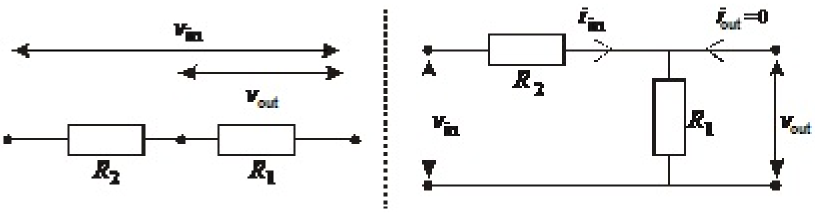


Figure 15 Voltage divider

A similar configuration is commonly used. The transfer function of this network is:

$$\frac{v_{out}}{v_{in}} = \frac{R_1}{R_1 + R_2} \qquad (11.)$$

The output voltage is always smaller than the input voltage, which explains the name "voltage divider". The expression is only valid when there is no load acting on the circuit: there can be no output current. If there was, however, a resistor connected to the output, this resistor would have to be taken in to account by considering it parallel to the R_1.

This expression is so commonly used that it is handy to know it by heart. If we, for example, connect a voltage generator with internal resistance R_2 to a voltage meter (oscilloscope) with input resistance R_1, we – unintentionally – create a voltage divider as given in equation (11.), which causes an error in the load. This error is relatively easy to estimate. In general, this error is desirably as small as possible. From this, we can reason that R_1 has to have a large value relative to R_2. If R_1 had a relatively large value, one would approach the transfer function of the voltage divider as:

$$\frac{v_{out}}{v_{in}} = \frac{1}{1+\frac{R_2}{R_1}} \approx 1 - \frac{R_2}{R_1} \qquad (12.)$$

The (relative) error due to the load is approximately the ratio between the internal resistance of the source and the load resistance. This rule of thumb can be used to quickly determine whether the error in connecting the measuring equipment can be neglected.
The formula for the voltage divider also holds for impedances (capacitances, inductances or combinations of those). In general, the resistances R_1 and R_2 can be replaced by impedances Z_1 and Z_2. Now the following applies for the complex amplitudes V_{in} and V_{out}:

$$\frac{V_{out}}{V_{in}} = \frac{Z_1}{Z_1 + Z_2} \qquad (13.)$$

In this relation, Z is the value of the impedance.
When the input voltage is a harmonic signal, there will be a change in amplitude, as well as in phase, due to this complex voltage divider. The relation between the out- and input amplitude A_{out}/A_{in}, the phase shift $\varphi=\varphi_{in}-\varphi_{out}$, and the relation between the complex amplitudes now become functions of the angular frequency $\omega = 2\pi f$.
One application of a complex voltage divider is the load of an oscilloscope. This is described in GLM §4.3.8. Before measuring with an oscilloscope, it is advised to study this paragraph.

Chapter 3

Physical construction and documentation of circuits

3.1 Hints for constructing circuits.

3.1.1 General suggestions

The construction of an electric circuit has to be done systematically. First you should make a measurement diagram on paper. This is a network model in which the sources and measurement devices are included. Mark in this diagram the node, which you define as the common node. This node is from now on called the system ground. All voltages in the circuit are defined relative to this node, which also means that the potential of the system ground is by definition **zero** volts.

Measurement of voltages, between nodes of which neither is at ground potential is only possible if one uses a floating measuring device (i.e. none of the measurement leads is connected to ground). One of the leads of an oscilloscope is often connected to ground, so only the voltage between the system ground and another node can be measured.
If we want to measure the current flowing through a branch of the circuit, we have to break the circuit and connect the ammeter in between.

Always make sure your setup is neat, this will make it much easier to find mistakes in your wiring. Try making use of different colors of wire, for instance use the same color for all wires connected to the same node.

3.1.2 Grounding

Grounding

Mains cables of the available measurement devices (the oscilloscope, multimeter and the function generator) all use a grounded mains cable, which is connected to the **ground clip** in the wall socket. The voltage between the ground clip and the soil beneath our feet is as close to zero volts as possible. The casing of these devices is connected to the earth in this way. The connection between the case and the earth protects the users from defects in the devices or in the mains wiring.

System ground and ground

There are complex electrical systems which consist of many different circuits, which all have their own system ground. In such a case it is important to make sure that all these system grounds have the same potential. If differences occur it can cause unwanted behaviour, or even damage. Therefore every system ground has to be connected to the ground-clips or a grounded measurement device.

Grounding through the oscilloscope, function generator and the multimeter

The oscilloscope

One of many ways to ground a circuit is to connect it to an oscilloscope; the outer layer of the BNC-connector is always grounded. Therefore an oscilloscope can only measure voltages relative to ground. Mind that the grounding is vanished once you disconnect the oscilloscope. Sometimes a circuit functions when being measured but stops working once the oscilloscope is disconnected.

It should be clear that grounding in this way is ill-advised. In circuits which are going to be used in a product, attention has to be paid to grounding that is not dependent on measurement devices.

The function generator and the multimeter

The HP-function generators and digital multimeters are isolated from the earth, so that floating circuits can be measured, but this isolation is not perfect. This is modelled by the connection between the “common” in- or output of the device and the case. This connection consists of a huge resistance and a small capacitance in parallel. See the manual of the function generator, page 282. Consequently, both instruments are effectively grounded at high frequencies.

Interference caused by ground

The mains ground wiring can cause disturbances and measurement errors. This is particularly an issue when the system ground and the mains ground are not connected. These disturbances are often hard to prevent. Therefore we give a few causes of these disturbances, as well as a few possible solutions.

Ground currents

The different devices are connected through a common system ground. Relatively large currents can flow through these connections (e.g. from an audio power amplifier or a power supply). The ground wiring has a non-zero resistance, meaning that not all “ground” nodes actually have the same potential. This can be remedied by:

- Short, thick wiring for ground connections (less resistance),
- Creating a star ground, all ground connections in a circuit should be connected to one single point (the current sink), instead of being connected to other devices before reaching the current sink. This is especially important on a breadboard, in places where the most sensitive components of the system are located.

Ground loops

In a loop of wire through which a changing magnetic field is applied, an induction voltage is generated. The bigger this loop, the higher the voltage. Because all ground wiring is electrically connected, it is very well possible that there is a loop somewhere. Furthermore, a changing magnetic field is always to be expected, for instance caused by a transformer in a power supply. This induced voltage is in series with the ground connection. Therefore the situation might occur, that not all ground nodes are at the same potential.

Possible remedies:

- Avoid ground loops as much as possible,
- Use shorter connections,
- Decrease the surface area of the loop by twisting the wires; this last remedy is often used with thinner cables connecting to a power supply.

3.1.3 The isolation transformer

As described before, the oscilloscope is always connected to mains ground, consequently also the BNC-cables connected to the oscilloscope will be grounded. Moreover we have seen that the function generator is also effectively grounded at high frequencies. This can cause issues in the circuit in which both instruments are used, because more than one point will be grounded in that circuit. In figure 1 there is no voltage over the lowest resistor, because both its terminals are grounded. This is obviously not intended.

To prevent these kind of problems, one can use the isolation transformer. Transformers will be discussed briefly in the lectures. For now it is enough to know that an isolation transformer is a box which reproduces the same **AC** voltage as it is fed. The input and output however, are galvanically isolated. Even though there is a grounding on the input side, the output is still floating. In figure 1 on the right-hand side there are now two current loops, which are each grounded independently. The voltage V on the left-hand side of the transformer is the same as the voltage V on the right-hand side of the transformer.

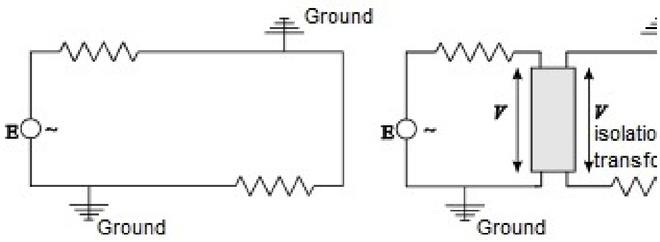


Figure 1 Left: Circuit with two grounds
Right: Two circuits with each one ground, but isolated using an isolation transformer.

3.2 Documentation of circuits

Wires

A schematic could be considered an idealized rendering of a circuit. Wires are also shown in a schematic, however this sometimes causes ambiguity. A **node** of wires can be rendered as a node with diverging wires, or as a wire with different branches. Of course the voltage is the same everywhere in this node, because wires in a schematic have no resistance.

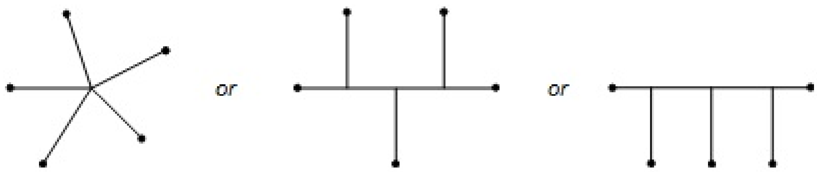


Figure 2 Nodes

In a schematic one could find crossing wires, which do not represent an electrical connection. To keep the schematic simple, the crossing wires are just represented by crossing wires. See figure 3. That is why one **should not** draw a node as two crossing wires, but as shown in figure 2. A node does represent an electrical connection, isn't it? Drawing a dot to signify crossing wires should be avoided as well.

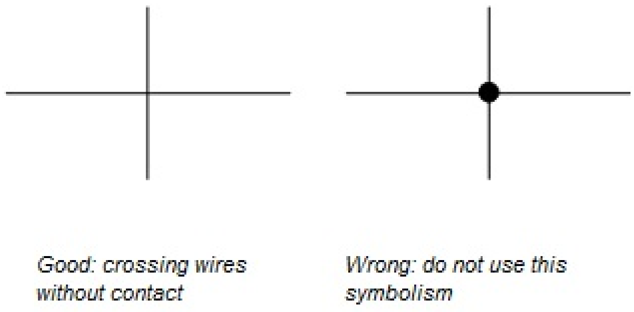


Figure 3 crossing wires

Measurement sketch

To measure the characteristics of an electrical circuit, one has to connect measurement devices, such as a power supply, an ammeter or an oscilloscope. This has to be done in the correct manner, but even if connected correctly, measurement devices could affect the circuit's behaviour. Even a wire and especially a coax cable (see below), can introduce additional measurement errors. To analyse these possible effects, it is useful to make a sketch which not only shows the circuit under test, but also the measurement devices which are connected. This enables one to find possible bugs more efficiently, this in contrast to just sifting through an unstructured bunch of wires. The first step in this process is drawing a "measurement sketch" (figure 4).

Such a schematic is meant to give a quick yet full overview of the most relevant elements in the experimental circuit. Central to this schematic are the components in the circuit under test, depicted with their usual symbols (the part in the dotted box). All measurement equipment used will be drawn - somewhat schematically - in rectangular box, with a label describing the nature of the device, e.g. an 'A' for an ammeter, a 'V' for a voltmeter, and 'OSC' for an oscilloscope. For an unambiguous interpretation, one should pay attention to the following:

- What is part of the circuit under test, and what are the measurement instruments; Draw for example a curvy line for measurement cables, and a straight line with a dotted lines around it for a BNC cable.
- At which nodes the instruments are connected to the circuit under test.
- At the nodes the circuit is grounded, one should draw the ground symbol: 

A BNC-cable, like the one used during the lab exercises, comprises an inner conductor with an insulating layer around it, and around that a flexible metal shield, which acts as the second wire. The outer metal conductor is often grounded, in this way it shields the inner conductor from unwanted disturbances. Between both conductors is a capacitance of around 100 pF/m.

Equivalent circuit diagram of the measuring circuit

To map the influences of the different measurement devices, we translate the measurement sketch to an equivalent circuit diagram. This schematic takes into account all relevant properties of the measurement devices, such as the internal impedance of an ammeter, a voltmeter, or an oscilloscope, but also the capacitance of coax cables.

During DC-measurements, the influence of an instrument can often be modeled with just a resistance (figure 4). With a multimeter being used as an voltmeter the impedance is extremely high (but not infinite), while when used as an ammeter the internal impedance is as low as possible (but still existent). The exact specifications of the instruments being used are written down in their manuals, which can be found on blackboard.

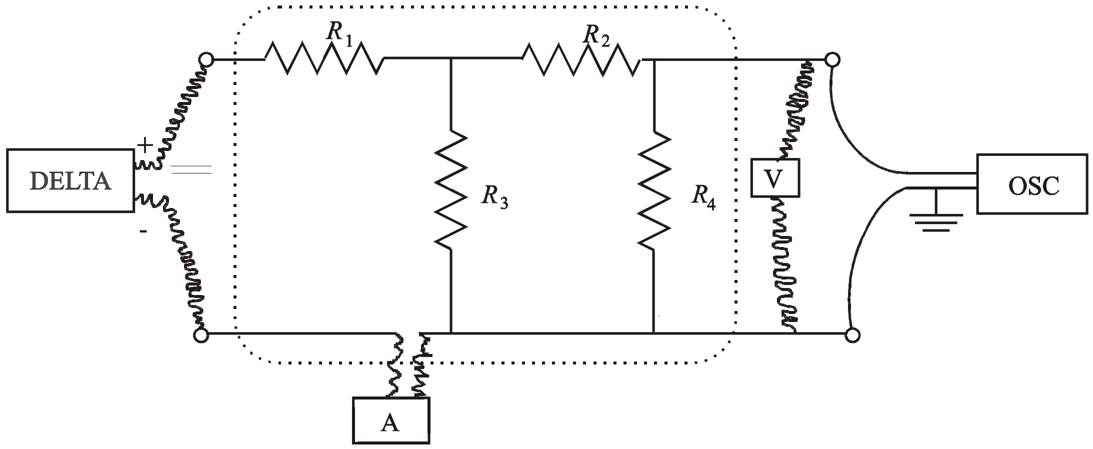


Figure 4 A measurement sketch

With AC measurements the internal impedance can often be modeled with a resistor and a capacitor in parallel. Note that a BNC cable has a capacitance which is parallel to the internal capacitance, and the equivalent capacitance is the sum of the two. An equivalent circuit diagram becomes rather complicated quickly with these parasitic capacitors, so make sure that a capacitor has a significant influence on the circuit before adding it to the equivalent circuit diagram.

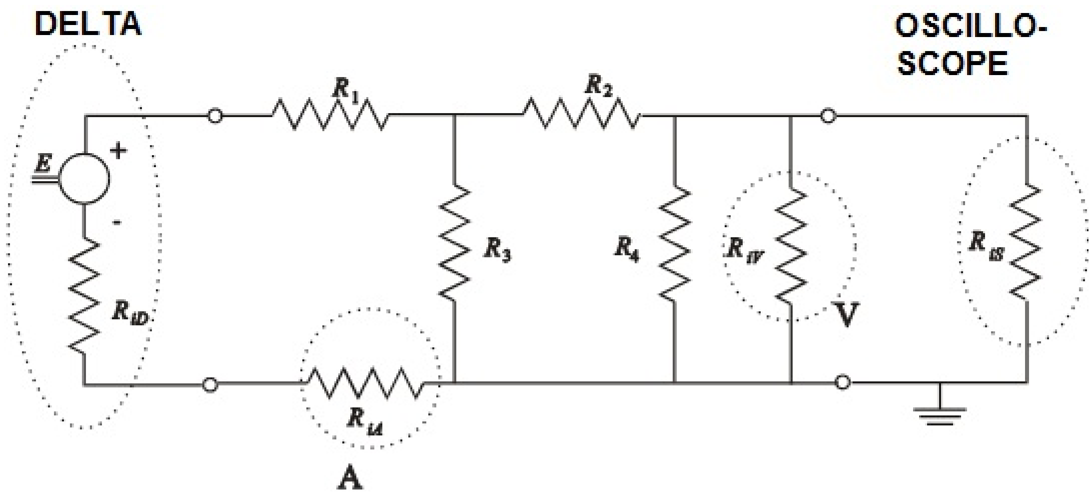


Figure 5 DC measurement circuit with modelled measurement devices

Chapter 4

Bridge circuits

4.1 Introduction

Bridge circuits are very suitable for precise measurements of impedances (or individual resistors, capacitors and inductors). The high accuracy is the result of a form of compensation: errors in the source and the detector are reduced and sometimes even eliminated.
The general bridge circuit is displayed in figure 1.

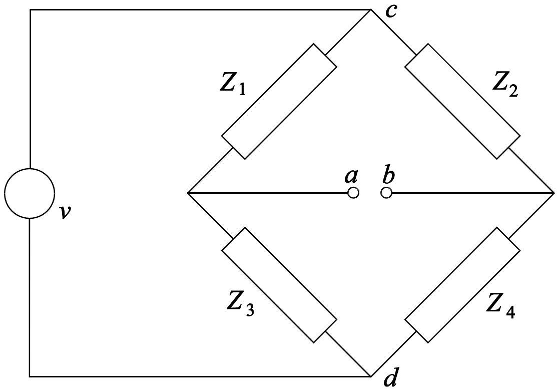


Figure 1 The principle of a bridge circuit

There are two ways to use a bridge circuit:

1. Measurement using the null method (full compensation);
2. Measurement using the indication method (partial compensation);

When using the null method, the voltage across points *a* and *b* will be zero under certain conditions: the bridge is balanced. One or two of the four impedances (Z_1 to Z_4) are variable. They need to be adjusted until equilibrium is reached. The equilibrium equation can then be used to determine the unknown impedances or other variables, such as a frequency, provided that the remaining impedances are known.

Properties of measurements using a bridge circuit

The big advantage of using bridge circuits is their precision. There are three reasons for this:

1. The condition for equilibrium is independent of the bridge voltage v_{cd}. Inaccuracies in the supply voltage will not be found in the measuring result.
2. The bridge's output voltage will be zero when the bridge is balanced. Determining a voltage of zero can be done with much greater accuracy than determining any arbitrary non-zero value.
3. There will be no voltage difference across the measurement instrument when the bridge is in equilibrium: its internal resistance will not cause any problem.

Bridge types

You are completely free to choose your own components when assembling the bridge. Every choice creates a different *bridge type*. The number of bridge types is therefore very large. The next section will discuss the Wheatstone bridge only.

4.2 The Wheatstone bridge

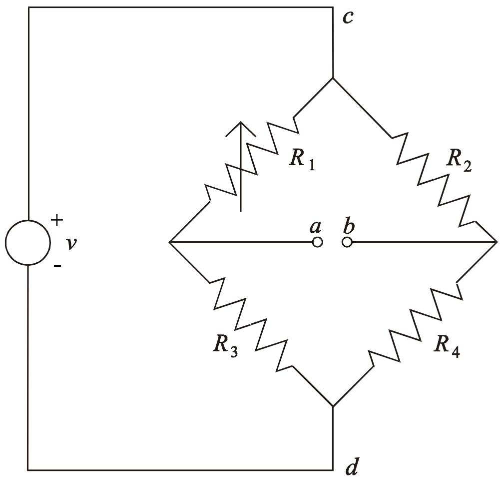


Figure 2 Wheatstone bridge

A Wheatstone bridge (displayed in figure 2) is used for the precise measurement of an unknown resistance. This is the easiest type: all impedances are resistors. The bridge's supply is a DC voltage v. Also see Nilsson, chapter 3.6.

Derivation of the condition of equilibrium

The bridge circuit in figure 2 can be interpreted as two voltage dividers. R_1 and R_3 form a voltage divider, in which:

$$v_{ad} = v_{cd}\left(\frac{R_3}{R_1+R_3}\right) \tag{1.}$$

Likewise, R_2 and R_4 form a voltage divider, in which:

$$v_{bd} = v_{cd}\left(\frac{R_4}{R_2+R_4}\right) \tag{2.}$$

The bridge's output voltage is equal to $v_{ab} = v_a - v_b$ (a differential voltage). It is equal to:

$$v_{ab} = v_{cd}\left(\frac{R_3}{R_1+R_3} - \frac{R_4}{R_2+R_4}\right) \tag{3.}$$

The bridge is balanced when $v_{ab} = 0$, i.e. when:

$$R_1R_4 = R_2R_3 \tag{4.}$$

This is called the *equilibrium equation*. When three resistors have known values, the remaining value can be calculated using the last equation.
The bridge can also be supplied with an AC voltage, instead of a DC voltage. Voltage v_{ab} will also be an AC voltage then. Because the Wheatstone bridge consists of resistors only, the condition of equilibrium will not change. It is easier to get an AC voltage to zero, than it is for a DC voltage. This is why precise measurements are usually done using an AC voltage.

Use of the null method

When we use the null method (mentioned in the introduction of this chapter), we will choose a fixed value for resistors R_3 and R_4 and use a variable resistor as R_1. We balance the bridge by varying R_1. (The arrow drawn on R_1 shows that R_1 is variable.) This way, we can calculate the value for R_2 from (4.):

$$R_2 = \frac{R_1R_4}{R_3} \tag{5.}$$

Use of the indication method

When we work using the indication method (mentioned in the introduction), we determine the unknown value for resistor R_2 from the measured voltage using equation (3.). The bridge is balanced ($v_{ab} = 0$) when all resistors have the same value. Now suppose that:

$$R_1 = R_3 = R_4 = R \tag{6.}$$

While R_2 differs a little from R:

$$R_2 = R + \Delta R \tag{7.}$$

It follows that:

$$\frac{v_{ab}}{v_{cd}} = \frac{1}{2} - \frac{R}{2R + \Delta R} = \frac{1}{2} - \frac{1}{2 + \frac{\Delta R}{R}} \approx \frac{\Delta R}{4R} \tag{8.}$$

The last similarity (≈) is obtained by using a Taylor series (in variable $\Delta R/R$).
We can see that the deviation ΔR can be directly obtained from the measured voltage v_{ab}. The drawback compared to the null method is that we have to take into account the finite internal resistance of the voltmeter. However, when the bridge is close to equilibrium, as is the case here, these effects can be neglected.

The use of strain gauges in experiment 2

In experiment 2, some weights will be measured using strain gauges, *i.e.* devices for actively measuring strain making use of the change in resistance. Both sides of the metal strip have a small circle with a horizontal and a vertical strain gauge in it. See figure 2 of experiment 2. Hence there are a total of four strain gauges, two horizontal and two vertical.

The vertical strain gauges

The vertical strain gauges are used as R_2 and R_3 in the Wheatstone bridge. Initially R_1 and R_4 are reference resistors. The following is true:

$$R_1 = R_4 = R \tag{9.}$$

When the strip is not strained, the strain gauges will have the same resistance as the reference resistors. Hence the bridge is balanced:

$$R_2 = R_3 = R$$

When the strip is strained, the resistance of the vertical strain gauges will be higher:

$$R_2 = R_3 = R + \Delta R \tag{10.}$$

This means the following is true:

$$\frac{v_{ab}}{v_{cd}} = \frac{R + \Delta R}{2R + \Delta R} - \frac{R}{2R + \Delta R} = \frac{\Delta R}{2R + \Delta R} \approx \frac{\Delta R}{2R} \tag{11.}$$

The acquired voltage has doubled compared to equation (8), thanks to the use of two vertical strain gauges. This makes it possible to determine ΔR accurately. This is why two vertical strain gauges are used, instead of one.

Compensation for temperature fluctuations using horizontal strain gauges

The previous section stated that the reference resistors have the same resistance as the strain gauges at rest. However, this is difficult to achieve in practice. The reference resistors are more or less constant, while the resistance of the strain gauges can vary as a result of temperature fluctuations. It is possible to replace the reference resistors by horizontal strain gauges as compensation. When straining the strip, the horizontal strain gauges do not stretch, so their resistance remains unchanged. Their resistance, however, does change as the result of changes in temperature. The unstrained strip is therefore always balanced, independent of the temperature.

4.3 Errors and sensitivities

Bridge circuits are very popular due to their great accuracy. For optimal use of the bridge, the following two conditions have to be met:

1. The bridge has to be designed in the way in which the measurements are as accurately as possible (sensitivity analysis).
2. We would like to have an impression of the accuracy that is achieved (error analysis).

Sensitivity analysis

In 4.2, an example was given for the use of a bridge for measuring a resistor. We would like to get an impression of the accuracy of these measurements. The reason why we could not have determined the variables, mentioned above, exactly, is the bridge voltage v_{ab} which cannot be set to exactly zero. This is because of the limited accuracy and resolution of the variable resistor and the voltmeter.
In the next section we will discuss the height of the measurement error as the result of the residual voltage v_{ab}. Suppose that we want to determine the impedance a of one of the components in the bridge. The bridge voltage is then dependent on this value, so $v_{ab} = f(a)$. The degree of dependency of v_{ab} on a is called the bridge's sensitivity. It is defined as the (first) derivative of v_{ab} to a. Hence,

$$\alpha_a = \frac{\partial\left(v_{ab} / v_{cd}\right)}{\partial a} \tag{12.}$$

(Notice that by dividing by v_{cd}, a has become independent of the supplied voltage.)
An error in v_{ab} (i.e. $v_{ab} \neq 0$ V) therefore results in an error in parameter a. Notice that when the sensitivity is higher, v_{ab} varies more and the bridge can be adjusted more precisely.
The error in parameter a will be called Δa; the error in v_{ab}/v_{cd} is denoted as $\Delta(v_{ab}/v_{cd})$.
From (12.), it follows that:

$$\Delta\left(v_{ab} / v_{cd}\right) = \alpha_a . \Delta a \tag{13.}$$

From which we can estimate Δa.
This is a simple example of the propagation of errors: the measurement error in v_{ab} causes a deviation in the calculated parameter a. We assumed that all the components' values are known accurately. When this is not the case, these deviations will also cause errors in the estimated value of a. A more general discussion about the propagation of errors can be found in the GLM, chapter 3.

Optimization of the bridge's settings

It is possible to choose components for an optimal accuracy of the estimated value of a. This means that a is at its maximum. The accuracy is optimal when $|\alpha_a|$ is as high as possible, see (13.). The values of the components, for maximum sensitivity of the bridge, are found by zeroing the derivative of the sensitivity α_a to the components' values x_i:

$$\frac{\partial \alpha_a}{\partial x_i} = 0 \tag{14.}$$

This equation is true for every component x_i. We therefore get as many equations as there are components in the bridge. A prerequisite for determining the derivation is that the bridge remains balanced when varying the values of the components. We will clarify this using Wheatstone's bridge.

Wheatstone's bridge

The parameter a to be measured here is resistor R_2. We will therefore calculate the optimal sensitivity of the bridge for estimating the value of this resistor and will take a look at the accuracy that is achieved.

Sensitivity analysis

The voltage v_{ab} is given by (3.).
The bridge's sensitivity to variations in R_2 is:

$$\alpha_{R_2} = \frac{\partial\left(v_{ab}/v_{cd}\right)}{\partial R_2} = \frac{R_4}{\left(R_2 + R_4\right)^2} \quad \left[\Omega^{-1}\right] \tag{15.}$$

In (15.) only one of the adjustable parameters is found: R_4. Differentiation with respect to R_4 and zeroing of the derivative provides:

$$R_4 = R_2 \tag{16.}$$

The sensitivity reduces to:

$$\alpha_{R_2} = \frac{1}{4R_2} \tag{17.}$$

So, for optimal sensitivity, R_2 and R_4 have to be equal. We do not know the value of R_2, so we have to pick a value for R_4 that is close to the expected value of R_2.
As stated before, the other components, R_1 and R_3, do not appear in the sensitivity equation. The bridge has to be balanced, though, so if we fill in (16.) in (4.) (the bridge's balance), it turns out that R_1 and R_3 also have to be equal:

$$R_1 = R_3 \tag{18.}$$

Error analysis

Filling in of (17.) in (13.) provides us with the error in the estimation of R_2:

$$\Delta R_2 = 4R_2.\Delta\left(v_{ab}/v_{cd}\right) \quad [\Omega] \tag{19.}$$

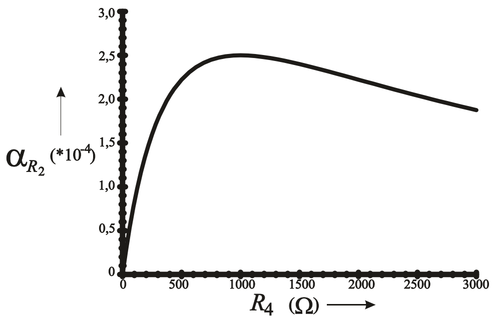


Figure 3 Sensitivity α as a function of the resistance R_4 (Wheatstone bridge)

Example

We have to measure the value of a resistor R_2 (of about 1000 Ω) using the Wheatstone bridge. The bridge's sensitivity to variations in the value of R_2 is obtained from (15.) and is displayed in figure 3, as a function of R_4.
We will adjust the bridge so that the sensitivity is a maximum. The graph shows that we have to pick a resistance R_4 of 1000 Ω. This is also shown in (16.).
Now we will estimate the accuracy of the measurement of R_2 that has to be done. We will assume a supply voltage v of 5V and a maximum residual voltage v_{ab} of 1 mV. The latter means that a v_{ab} of more than 1 mV will be noticed and that the bridge's settings will be adjusted in this case.

From (19.) it follows that the error in the measured value of R_2 is equal to 0.8 Ω.
Mind you, the numbers used above are merely examples.

Chapter 5

Description of the measurement equipment

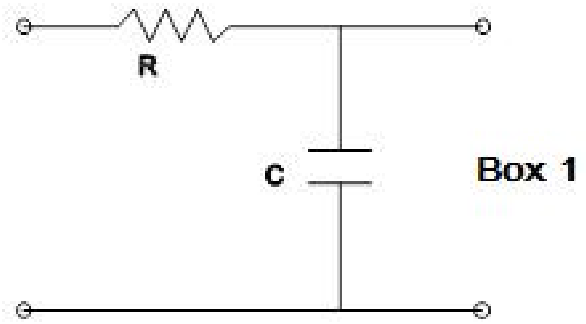
5.1 Description

The equipment available in the lab is described in the manuals supplied by their manufacturers. These manuals can be found on the Blackboard website.
The following equipment is at your disposal:

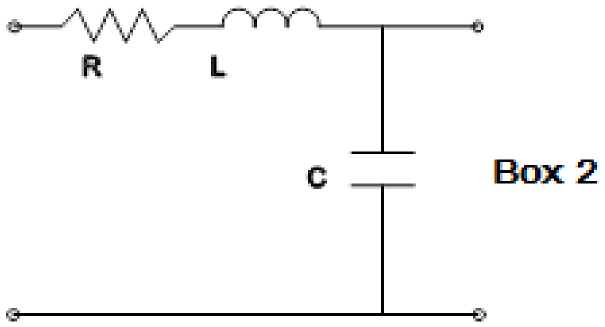
1. Tektronix TDS 2022B digital oscilloscope,
2. Hewlett Packard 33120A or Agilent 33220A function generator,
3. Delta power supply,
4. AC/DC adapter,
5. Hewlett Packard 34401A digital multimeter (the Agilent 34401A is identical),
6. Tektronix P 2220 probe.

In addition, a few boxes containing small circuits have been made, which will be used during some of the experiments. These are:

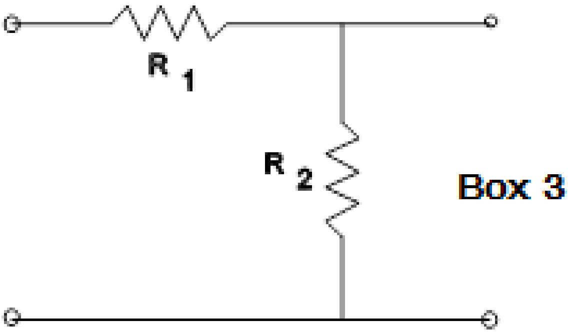
7. A precision, measuring resistor of 10 Ω (1%),
8. A precision, measuring resistor of 1 MΩ (1%),
9. An RC-circuit,



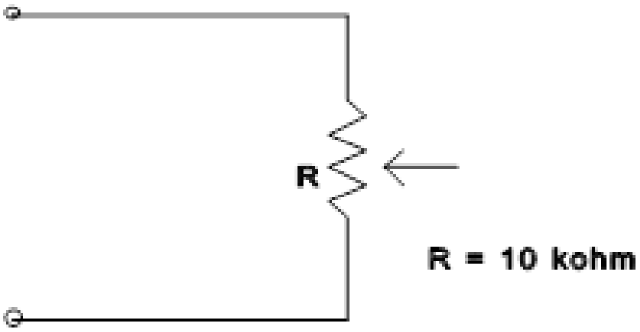
10. An RLC-circuit,



11. An RR-circuit,



12. A potentiometer (variable resistor),



13. A breadboard containing a bridge circuit,
14. A bridge circuit containing strain gauges,
15. A measuring transformer,
16. A box used in Fourier analysis.

Experiment 1

The diode

Introduction

Of all passive components, such as resistors, inductors, capacitors and the like, the diode may have the most remarkable properties. Its resistance is almost zero for currents in the *forward* direction. Currents in the *reverse* direction experience an almost infinite resistance. This means that we have to be careful when characterising the diode. In part 1 of this experiment we will measure its current to voltage behaviour using DC voltages. We will also look for the best way to plot our results in a graph. In part 2 we will try to measure the diode's behaviour using the oscilloscope. This is useful, but does have some limitations.

Part I - Measuring the behaviour of a diode using DC voltages

Introduction

Definition of the problem

Measuring the behaviour of components using a voltmeter and an ammeter. Electric circuits are used to turn one electrical signal into another electrical signal. A resistor will turn a voltage into a current, for example. The behaviour of a circuit can be described by plotting the output signal against the input signal in a graph. In this way we obtain the device characteristics in static mode. This experiment will pay attention to the method for measuring this behaviour. The device under this case study will be a diode.

Analysis

The diode

An ideal diode is a switching element that lets through current in one direction (R=0) and fully blocks currents in the reverse direction (R is infinite). Diodes are commonly used for rectifying alternating currents. In practice, a diode does have a small resistance in the forward direction and its resistance in the reverse direction is very high, but not infinite. The relationship between the applied voltage and the current that is let through, is called the diode characteristics. It is given by:

$$i(v) = I_0 (e^{av} - 1) \qquad (1.)$$

Herein, v is the applied voltage, i the acquired current and a and I_0 are two parameters, describing the behaviour of the diode. The current increases rapidly with the voltage in the forward direction, while it quickly adopts the (small) saturation value $-I_0$ in the reverse direction.

Methods

Analysing the diode characteristic

We would like to find out whether the diode will behave according to the theory, that is, whether it will behave according to the exponential relationship (1.). We would also like to determine the parameter a. An indication for the validity of the theory can easily be obtained by a graph, in which the diode characteristic curve is drawn cleverly. First, read the GLM, chapter 2.3, about the different kinds of graphs.

1 Show that the relationship given in (1.) can be rewritten to:

$$\ln(i + I_0) = av + \ln(I_0) \qquad (2.)$$

2 What kind of graph do you prefer for displaying the diode characteristics; in particular, what kind do you prefer when you want to determine the value of a?

Measuring circuit

Design circuits for determining the diode characteristics. Draw the measuring circuits.

Note that the resistance of the diode is very low in the *forward* direction. Pick a measuring circuit that does not produce errors due to the measurement instruments.

Note that the resistance of the diode is very high in the reverse direction. Measurements of the currents therefore have to be done using a measuring resistor (already used during the IEEE lab course). Assuming an expected current in the range of nanoamperes, what value is required for the measurement resistor?

Execution

Build the designed circuit for measurements in the forward direction. Write down the internal resistance of the instruments along with their range and type. Measure values in the forward direction. Let i vary between 0 and 5 mA. Draw the characteristic curve using the type of graph you found in question 2*. Give an estimation of a and I_0.
Build the designed circuit for measurements in the reverse direction and measure a number of values in the reverse direction. Let the voltage over the diode vary between 0 and 10V.

Conclusions

3* Does the current of the diode satisfy equation (1.) in the forward direction?
4* Is the value of the current in the reverse direction what you expected?
5* Did the measuring resistor, used in the reverse direction, have the desired value?

Part II - Measuring the diode's characteristics using the oscilloscope

Introduction

Definition of the problem

Measuring the characteristic curve of a component using the oscilloscope.

- Measuring and analysing the characteristic curve of a diode

We are now going to put the characteristics of the diode on an oscilloscope. Afterwards, we can compare the results of part 1 (current-voltage measurements) and part 2 (measurements using the oscilloscope).

Analysis

The diode

The behaviour of the diode has already been discussed in part 1.

Measuring the diode's characteristics

This time, the diode's characteristics will be put on the oscilloscope as a whole (*i.e.* the dynamic method). To do this, we will use the X-Y mode of the oscilloscope. You might want to read GLM chapter 4.3.3 and 4.3.8 (only the beginning, Lissajous images will be introduced later in this course) for information about the X-t and X-Y mode.

Questions

6a Please explain the chosen measurement setup (scope settings, offset) and what signal (frequency, shape wave) is used. What to expect from this setup regarding the potential distribution? What do you expect?

6 Give advantages and disadvantages of the static method and the dynamic method.

Methods

Grounding

The BNC output of the digital HP 33120A and Agilent 33220A function generators is, in contrast to the previously used analogue function generators, not grounded. Therefore, unless very high frequencies are used, the function generator will not introduce any grounding problems. *Both channels on the oscilloscope are grounded, though, and this might cause problems. When both channels are used simultaneously, like they will be in this exercise, the ground points need to coincide to prevent grounding problems.*

Measuring resistors

Study the GLM, chapter 2.3.3

Ingredients for designing a circuit

- The diode,
- A resistor for the indirect measurement of the current through the diode,
- A function generator,
- The oscilloscope.

Execution

- Set up the oscilloscope like you did in module 2 of the IEEE course.
- Make sure the current through the diode varies between 0.5 and 5 mA.
- Also draw the screen image of the oscilloscope.
- Make a graph of the diode's characteristics.
- Estimate parameter *a* from equation (1.)

Conclusions

7* Are both graphs, the one created with current-voltage measurements (part 1 in your journal) and the one made with the oscilloscope, in agreement? Why (not)?

8* Can the diode current be described according to the equation of part 1?

Experiment 2

The Wheatstone bridge

Introduction

During this experiment you will be encounter a measuring problem, namely determining the mass of a weight. We will do this using so called strain gauges. These are resistors that vary in resistance when they are stretched. The gauges themselves are fragile, so it's common to glue them onto surfaces to measure how much these surfaces stretch. Strain gauges don't stretch much so our goal is to measure very small variations in resistance. We can do this with the Wheatstone bridge.
First, we will be looking at the properties of the Wheatstone bridge. Then we will try to determine the mass of a weight using the Wheatstone bridge.

Equipment

In part I of this experiment we will be making a bridge out of standard components and examine the behaviour of the bridge. In part II we will try to determine weight using the strain gauges by using amplification circuits to increase the sensitivity. These circuits are available in a dedicated casing (see figure 4).

Error discussion

During part I we will discuss the error resulting from the accumulation of multiple smaller errors. During part II we will be looking at the behaviour of the inaccuracy as a function of properties of the circuit. You do not have to do actual numerical calculations for this step.

Part I - *The behavior of the Wheatstone bridge*

Introduction

Problem

You will examine the behaviour of the bridge: what is the equilibrium of the bridge and what the behaviour of the bridge voltage as a function of the resistor values.

Analysis

Please read chapter 4 of this manual. Skip the part about the strain gauges. They will be dealt with in part 2.
1 Give the circuit of the bridge.
2 Give the general equilibrium equation as a function of resistor values.
Now, assume that the resistors R_2, R_3 and R_4 are equal and R_1 has a slight difference ΔR.
3 Express the voltage v_{ab} as a function of ΔR.

4   The Wheatstone bridge can be used to accurately measure resistances. Assume R_2 to be unknown, R_3 and R_4 static and R_1 variable. Write down an expression for R_2 as a function of the other resistor values.
5   Determine the error in R_2 as a function of the error of the bridge voltage and resistor values (see chapter 4).

Method

Design a measurement setup that allows you to check the equilibrium equation of your bridge. Use the following components: Four resistors, a function generator (as an AC voltage source), a multimeter and a breadboard.
Use a variable resistor for R_1, also known as a potentiometer or potmeter. A common implementation of the potmeter is given below in figure 1. You can see that this potmeter has three terminals. The outer two, X and Z, are attached to the endings of the resistor. Terminal Y is placed somewhere in between. The resistance between terminals X and Y R_{XY} is proportional to the length l_{XY}. The resistance R_{YZ} is then the difference between R_{XY} and the total resistance R_{XZ}. Together this results in: $R_{XZ} = R_{XY} + R_{YZ}$ with $\frac{R_{XY}}{R_{YZ}} \propto \frac{l_{XY}}{l_{YZ}}$

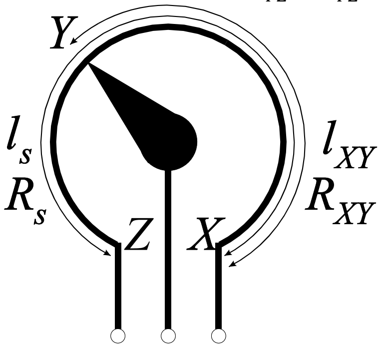


Figure 1. The schematic of a potmeter.

Execution

Use an AC signal set to 5V as voltage source. The reason for this is because the bridge is easier to calibrate to its equilibrium (in this case 0V) using AC signals. Use a frequency of about 100Hz. Try to avoid using frequencies around 50Hz, as this is the frequency of the line power, and may cause serious problems.
Pick three equal resistors for R_2, R_3 and R_4 with a value of about $3\text{k}\Omega$. The potmeter that you will be using provides you with resistance values between 0 and $10\text{k}\Omega$.
Now, configure your circuit to a bridge equilibrium. Then, vary the potmeter resistance with steps of 50Ω and measure the voltage across the bridge. Compare your results with the theory.
Make sure you measure the resistance of the potmeter with a multimeter. Also, make sure that you re-measure the resistance every time you detach your potmeter from the circuit.
Also, keep measuring the supply voltage with a voltmeter. Make sure the voltage does not drop.
Calculate the value of R_2 as a function of the other resistance values, when you accomplished a bridge equilibrium by varying R_1. Estimate the error of the measured bridge voltage (which is supposed to be 0). Then, calculate the error in R_2.

Conclusion

6*   Is the point of your bridge equilibrium correct?
7*   When the bridge is not in the equilibrium position, are the measured voltages across the bridge logical and according to theory?
8*   Does the calculated value of the resistance fall within the error range that you calculated earlier?

Part II - Using the Wheatstone bridge for weight measurement

Introduction

This experiment is the first that will be dealing with experimental measuring.
Measuring is such a common activity, that often we do not even realize it anymore. A few examples are: reading a barometer, a thermometer, the weighting scale at home or reading your speed gauge, odometer or fuel gauge in your car etc.
In all these cases we rarely question the accuracy of these measurements, the actual quantity which we are measuring or the validity of our measurement (is our measuring device calibrated?). We often assume that measuring equipment is providing us with the appropriate data, but this is not always the case.
The act of experimental measurement is different from these simple "household" measurements, because of the fact that experimental measurements are always interpreted critically and checked for validity. Here are some examples:

- Does the temperature influence the measurement?
- Does the act of measuring itself influence the actual measurement?
- Is the accuracy of the measuring device acceptable for this measurement?
- Does the measured value fall within the range of expected values? Is the result plausible?

We will be facing these questions during this experiment. We will look for causes of certain problems and try to find solutions for those.

An experimental measurement is almost always done with some sort of electrical device. The measured quantity is somehow converted to a measurable electrical quantity, for example voltage, current or electrical resistance. This conversion is done with a so called transducer. This is a device that converts physical quantities to a measurable electrical parameter. Common examples are microphones, solar cells or metal detectors.
This test is completely dedicated to measuring the mechanical strain and deformation of a metallic strip. Our goal in the first place, is not to measure these mechanical quantities, but to examine the problems that we might be facing during such measurements. These problems are not only common in these measurements but in all kinds of measurements. They are most often the result of limitations of the electrical measuring equipment. Later in this course, we will see how we can model these systems with electrical circuits. This will provide us with a method to describe these limitations, estimate their influence and, if necessary, correct or improve them.

Problem

We want to measure the mass of two weights. We will be doing this by hanging them on a metal strip and measuring the stretching of this strip. That stretch can be measured with strain gauges. These are thin, flat resistors that are glued onto the metal strip. The strain gauges then stretch proportionally to the metal strip. If the lengths of the strain gauges increased, the resistance would increase also. This change in resistance can be converted into our mass by using multiple equations.

Part II A

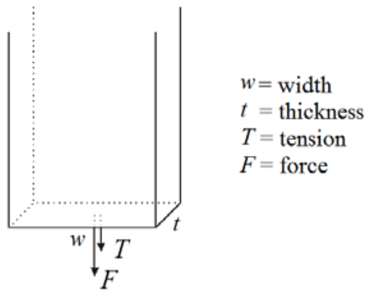


Figure 2 Force and tension on a metal strip

Analysis

The described measuring methods are used a lot in technical areas, for example in large constructions like bridges, cranes or airplane wings etc. You can also find them in weighting scales and all kinds of force and pressure transducers. The following principles are present in this experiment.

When a force F [N] is applied on a mass m [kg], there will be an acceleration a [m/s^2]:

$a = F / m$.

Near our planet gravity is forced upon every object. The acceleration g is roughly 9,8 m/s^2.

Tension T is the applied force per unit surface on the cross sectional area of the strip.

Stretch factor K is the quantity in which the resistance of the strain gauge changes, relatively to its original value, as a cause of stretching (also relative, but now to its original length, *i.e. strain*):

$$\frac{\Delta R}{R} = K \frac{\Delta l}{l}$$

Most metals give a stretch factor of about K=2.

Further, Young's modulus E is the amount of relative stretching or strain ($\Delta l / l$) that takes place as a result of mechanical tension or pressure T:

$$\frac{\Delta l}{l} = \frac{T}{E}$$

Brass has a Young's modulus of E=1,0.10^{11} N/m^2.

9 A strain gauge at rest has a resistance of 120Ω. Calculate the change in resistance if this strip is stretched to a 1% length increment.

10 The strain gauge is glued to a brass strip with a width of 15mm and a thickness of 0.5mm. We will be hanging a weight of 1 kg on this strip. The weight will be pulling in the longitudinal direction of the strip. Calculate the relative length change ($\Delta l / l$) or strain and the relative resistance change ($\Delta R / R$) of the strain gauge. Also give an equation for the mass *m* as a function of the measured resistance change ΔR.

Method

We only possess brass strips to which the strain gauges are attached. This strip will be hung vertically and thus can be loaded longitudinal. A strain gauge contains two separate strain gauges, one horizontally and one vertically. We will only be using the vertical one. The horizontal one will be used later on.

Determining the resistance change

We want to know the change in resistance ΔR, when the strip is stressed. There are two possibilities to do this.

Possibility 1:
Measure the resistance in rest condition (R_1).
Measure the resistance in stressed condition (R_2).
Determine the difference between the two conditions $\Delta R = R_1 - R_2$.

11* Do you think that ΔR can be calculated accurately, if you look at the values that you have calculated in 9*?
12* Is this a good way to calculate ΔR?

Possibility 2:
Since we are only interested in the change, we can also apply the Difference method: We measure directly what the difference is between the static resistance and the resistance that we want to measure. The resistance of the strain gauge in rest will be the resistance of the static (reference) resistor. By doing this, we basically change the reference voltage, so that we will only measure a value different from 0 when the strain gauge is stretched or compressed. Because of this we can further increase the length of the scale and measure small changes more accurately.

13* Reason why this is a better method than the one we described above in possibility 1?

Checking the theory and noise

We want to verify whether the analysis that we discussed is according to the theory and to what extent noise is playing a part in our measurement.
We are checking:

- **the stability:**
  This quantity indicates how much the measured value is fluctuating during our measurement; distinguish slow and fast changes and relate, as far as possible, the variety compared to the average value.
- **the reproducibility:**
  This is the quantity that indicates how much the measured values change, when we reproduce the same measurements. Relate how big this change is compared to your actual measured values. What is the order of magnitude?
- **the non-linearity:**
  This is the degree in which the force and resistance relation differ from a linear function. Define a quantity for this non-linearity and estimate the non-linearity of the measurements by your definition.

These checks can be executed by measuring the resistance change in the strain gauges several times for two weights; one of 1 kg and one of 2 kg.

The Wheatstone bridge

The Wheatstone bridge that you will be using is built into a casing together with an amplification circuit that will amplify the generated voltage by a factor of 1000. The bridge has to be put into equilibrium when the strain gauge is not stressed. Then, we have to measure the resulting voltage when the strain gauge is stressed. The difference will be called Δv.

14* Derive the fact that the difference in resistance of the strain gauge in stressed condition can be found to be according to

$$\frac{\Delta v}{v} = 500\frac{\Delta R}{R}$$

Where v is the **supply voltage** used inside the casing. (Please look at chapter 4 of this manual).

Execution

During the execution of the experiment, the values for the resistance, length and width etc. will be used as given in questions 9* and 10*. The supply voltage of the casing will be supplied by the block with the plug. Hook up the horizontal and vertical strain gauge to the bridge case according to the schematic below in figure 4.

Please note the way to do this is, is exactly the opposite of what you would expect. You can see this by looking at how the resistance wires are placed inside the strain gauge.

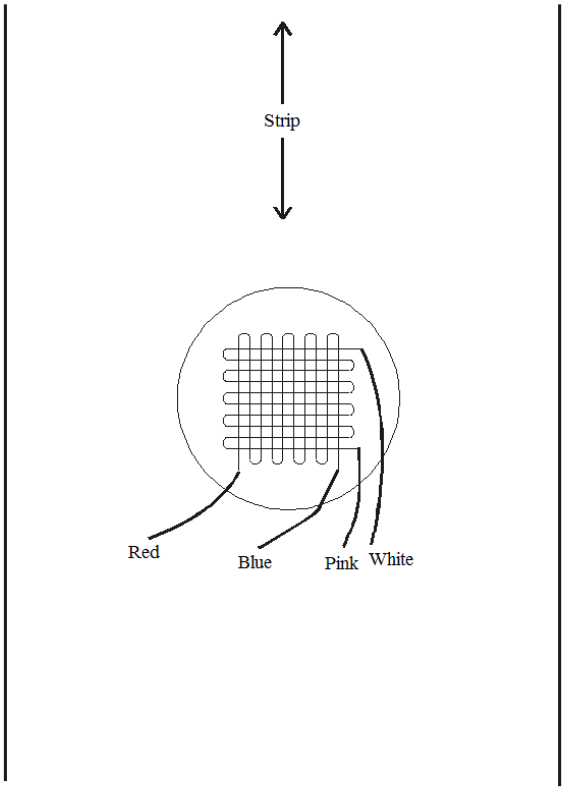


Figure 3 strain gauges on the metal strip

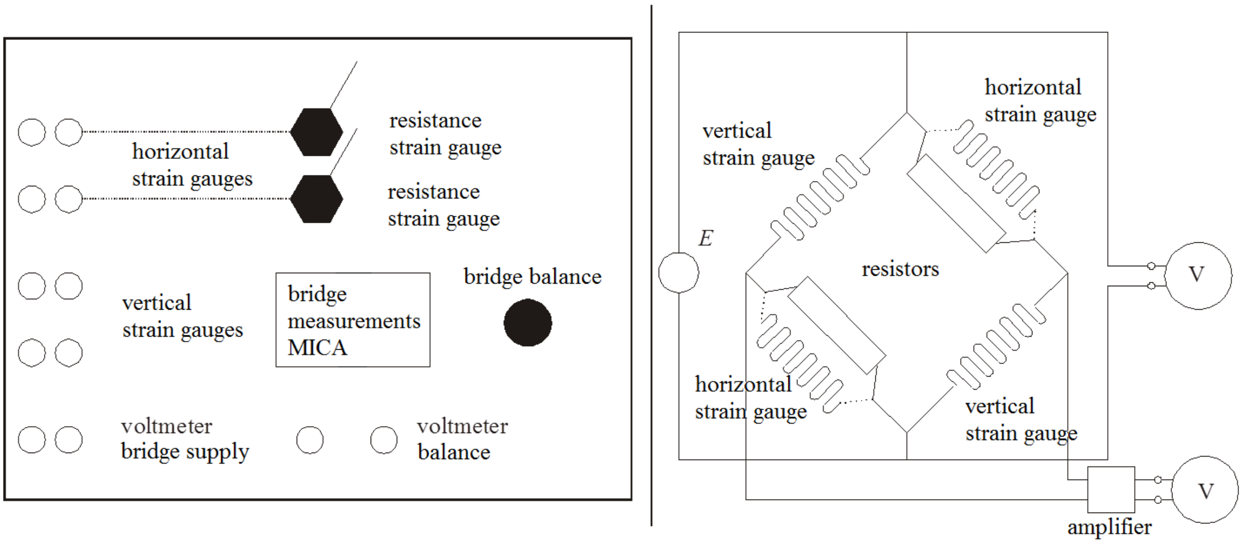


Figure 4 The casing of the Wheatstone bridge, left: layout, right: content

The casing is supplied with the black block. So don't supply it externally with other power supplies!

Measuring

Use the ***HP*** *or* ***Agilent*** *digital multimeters for the voltage measurements. Turn the switch "weerstand"/"rekstrook" to the "weerstand" mode.*

1. Use the “voltmeter brugvoeding” connectors to measure the power supply voltage V.

   Please note: Always measure DC voltages for all your measurements!

   Check your power supply voltage every time you measure. Make sure it does not drop.

2. Now connect a voltmeter to the connectors saying “voltmeter evenwicht”. Make sure the strain gauge is not stressed and read what the meter is indicating. If the gauge indicates anything besides 0, you should calibrate it using the “brugbalans” knob.
3. Measure the difference in resistance by applying one, and then two weights to the strain gauge.
4. Repeat this measurement once more. Write down the found values in a table.

Calculations

Calculate the mass of the weight by using the measured values for ΔR.

Conclusion

What can you conclude about:

15* the stability;
16* the reproducibility;
17* the non-linearity?

Part II B (Bonus) - An improved method using strain gauges

Introduction

Definition of the problem

During the previous measurements we have discovered a problem, namely, that the value indicated by the meter is not stable enough (right?). In part II B we will be looking at one of the causes of this phenomenon: the resistance of the strain gauge is influenced by the varying temperature. In this part, we will be looking at a measuring method that compensates for these temperature fluctuations.

Analysis

To compensate for the temperature fluctuations, we will have to replace the reference resistors, which we have been using. By using strain gauges instead, the temperature fluctuations will be cancelled out, because we are now compensating with only resistors that vary equally with the temperature. Please look at the description in chapter 4.

Method

We still use the same equipment as in part II A:

- Weights,
- Reference resistors,
- Strain gauges.

On each strip we put two circles. Each of them is comprises a vertical strain gauge, which we will use to measure the stretching, and a horizontal one. See figure 4 for the geometry of the used strain gauges.

The behavior of the strain gauges without a load

18* What should be the difference in resistance between the strain gauges if they are not stressed?

19* What should be the difference in resistance between the strain gauges if they are not stretched, but changing in temperature?

The behavior of the strain gauges with a load

If the strip is stretched in a certain direction, then it will shrink in the other direction. Suppose that a force F is applied in the vertical z-direction, resulting in a length increase Δl_z. The horizontal x-direction will have a length change $\Delta l_x / l_x$:

$$\frac{\Delta l_x}{l_x} = -\mu . \frac{\Delta l_z}{l_z}$$

Where μ is the Poisson constant.
For brass, the Poisson constant is 0.3.

Apparently, Δl_x is negative, as there is a minus sign on the right hand side of the equation, indicating a shrinking strain gauge. This results in a changing resistance for the horizontal strain gauges.

$$\frac{\Delta R_x}{R_x} = K \frac{\Delta l_x}{l_x}$$

with K being the same as in IIA.

Because Δl_x is negative, ΔR_x is also negative, or in other words: R_x will be smaller.

20* Give the voltage over the bridge as a function of resistance difference of both the horizontal and vertical strain gauges.

21* Give the relation between the voltage over the bridge and the weights applied to the strain gauges.

Execution

Repeat the same series of measurements as you did in part II A, but this time using the horizontal strain gauges as reference resistors. Use the equation that you defined above to process the data.

Conclusion

22* Which of the previous experiments gave you the best results for the mass of the weights?

23* Answer the question 12 until 14 but now for the experiment in Part II B. Is there any improvement?

Experiment 3

Response of first and second order circuits

Introduction

Definition of the problem

During this lab exercise we're going to put the theory of electric circuits about first and second order networks into practice. These circuits can be analysed in both the time and the frequency domain. In the time domain we're going to look into step responses (an instant change in voltage or current), for example the response of a square wave signal. In the frequency domain, we are going to look at the response as a function of frequencies. In this experiment we will look into the time-domain behaviour of RC, RL and RLC circuits. A later lab exercise will deal with frequency dependent behaviour.

Part I – first order circuits

First order circuits consist of a resistor and a capacitor (RC-circuit) or a resistor and an inductor (RL-circuit).

Analysis

The voltage $v(t)$ over a resistor-capacitor pair (figure 1) is equal to the sum of voltage $v_c(t)$ over the capacitor and the voltage $v_r(t)$ *over the resistor*:

$$v(t) = v_C(t) + v_R(t)$$

1 - Formulate a differential equation which describes this circuit.
2 - Give an expression for $v_R(t)$ and for $v_c(t)$ as a function of the time constant $\tau=RC$ and the given voltage $v(t)$. (Also give a description how you came up with this expression.)

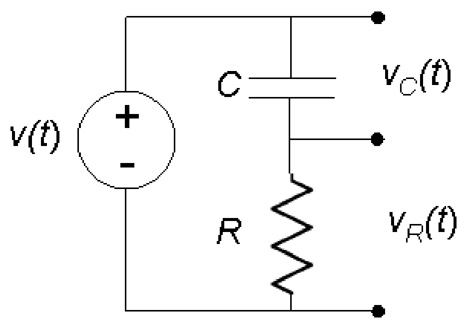


Figure 1 – RC circuit

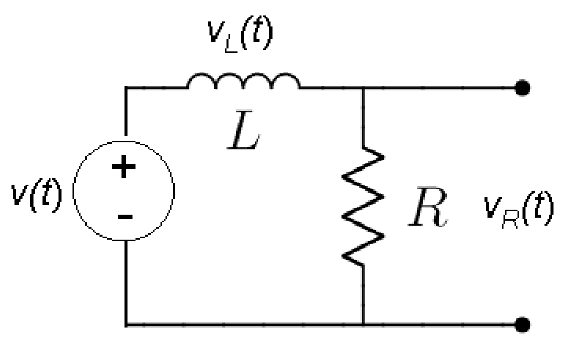


Figure 2 – RL circuit

An RL circuit is displayed in figure 2:

3 Give the differential equation for this circuit and give an expression for $v_r(t)$ and $v_L(t)$ with the time constant τ.

Method

Ingredients:

- Function generator,
- Oscilloscope,
- Resistors, capacitors and inductors (see text),
- Breadboard.

Execution

Measuring the voltage over a resistor for an RC circuit

First of all, we're going to measure the step response of the RC circuit. In order to measure this response, we will use a square wave as input signal. Obviously one has to choose the frequency of the square wave in such a way, which enables to observe the time constant of the RC circuit clearly. The voltages of the square wave have to be chosen correctly, for instance a negative voltage over the capacitor should never occur. Therefore we choose a square wave with V_{on}=5V and V_{off} = 0V

Use a capacitor of 0.33μF and a resistor of 1kΩ. Graph the step response using the oscilloscope.

Measurement of the voltage over the capacitor in the RC circuit

Also measure the voltage over the capacitor, plot it using the oscilloscope screen and determine the time constant using this plot.

Conclusion

4 Compare the time constant which you measured with the theoretical time constant. Are they in agreement? (*Hint: remember experiment 1 plotting the I-V characteristics of a diode).*
What can be said about the accuracy?

5 What does the integral of the step signal look like?
Which part of the measured step response represents the integrated input signal?

Method

Now we're going to take a look at the step response of an RL circuit. In the real world inductors, besides having a self-inductance L, have an internal ohmic resistance R_L. This resistance is of significant value. If we take the circuit of figure 2, we can write:

$$v(t) = \left[L\frac{di}{dt} + iR_L \right] + iR$$

The internal resistance is not always taken into account in the circuit schematics, but always has to be taken into account with calculations. This complicates the measurement, because you cannot simply measure the voltage over the inductance L without measuring the voltage over the parasitic resistance R_L. This resistance is not equal to the DC resistance of the inductor either, all sorts of effects constitute to this resistance in the end.

Execution

Now we are going to take an inductor with an unknown impedance, and determine its self-inductance and its internal ohmic resistance (ESR or Equivalent Series Resistance). Take an external resistor of 1kΩ. Build the RL circuit, plot the step response using the oscilloscope and determine the time constant.

Determine the self-inductance and the ESR of this unknown inductor using your own measurements. There are multiple ways of determining the ESR, i.e. by looking at the resulting voltage over resistor R.

To test your calculations and measurements, you could repeat this using a known inductor, e.g. that with 22mH inductance. In the lab are multiple inductors with datasheets at your disposal (via Blackboard).

Conclusion

6 Compare the calculated values with the measured ones using the RLC meter. Discuss the possible discrepancies between them and also the accuracy of both measurements.

7 Explain the differences between a relatively small 22mH inductor and the relatively large unknown inductor.

8 Is this measurement method suitable to measure inductors?

Part II – second order circuits

Analysis

The natural response of an RLC circuit is caused by the release or conservation of energy by a self-inductance and/or a capacitance. This occurs as a result of a sudden change in current or voltage in the circuit. The mathematical description of voltages and currents in RLC circuits are given by second order differential equations. These equations can be obtained by using Kirchhoff's law. This is the general form of such an equation:

$$\frac{d^2x}{dt^2} + 2\alpha\frac{dx}{dt} + \omega_0^2 x = C$$

With x being the voltage over a capacitance or the current through an inductance.
Therefore the characteristic equation is:

$$s^2 + 2\alpha s + \omega_0^2 = 0$$

With solutions:

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

The three possible responses which can occur are:

a) Overdamped, when $\alpha^2 > \omega_o^2$

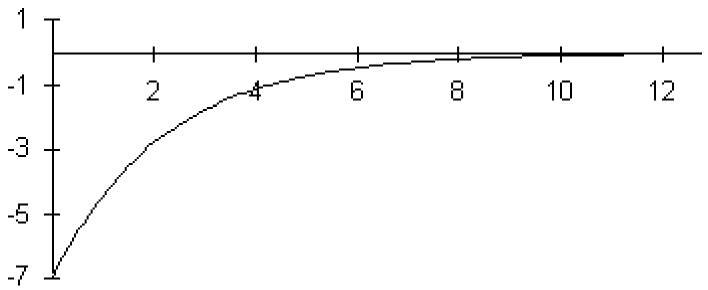


Figure 3

b) Critically damped, when $\alpha^2 = \omega_o^2$

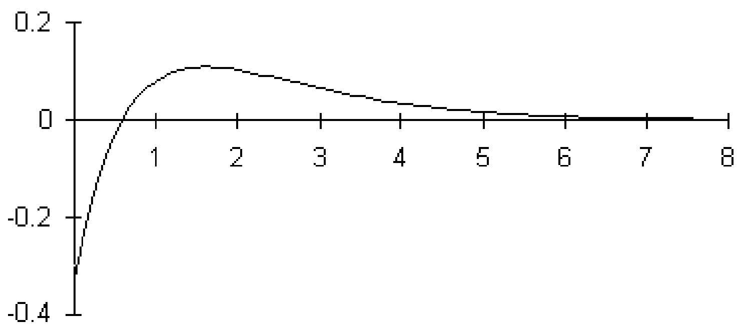


Figure 4

c) Under damped, when $\alpha^2 < \omega_o^2$

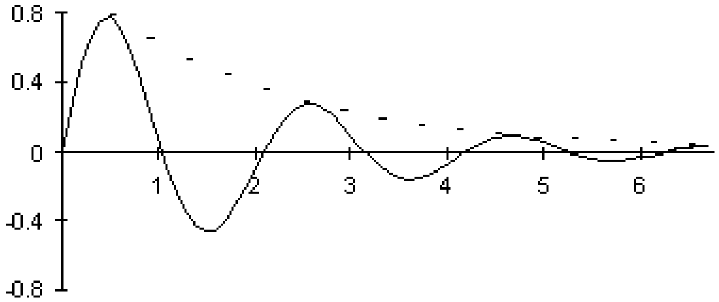


Figure 5

In this experiment we are going to further analyse and measure these responses.

The solutions to the responses are:

$$x_{over}(t) = K_1 e^{s_1 t} + K_2 e^{s_2 t}$$

$$x_{under}(t) = (K_1 \cos\omega_d t + K_2 \sin\omega_d t)e^{-\alpha t}, \omega_d = \sqrt{\omega_0^2 - \alpha^2}$$

$$x_{critical}(t) = (K_1 t + K_2)e^{-\alpha t}$$

If we want to feed an RLC circuit with a step function, we need to add the final value X_f (see table 8.4 in Nilsson).

This results in the following responses:

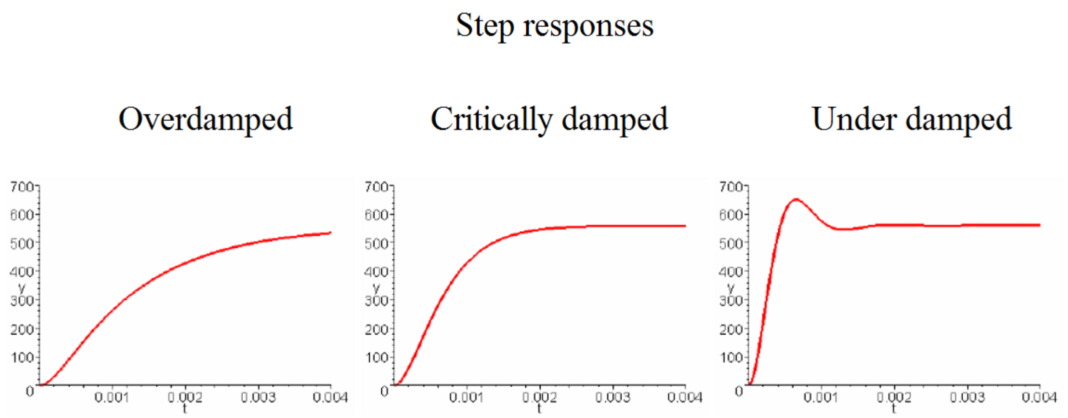


Figure 5 – step response

We are now going to build the circuit shown in figure 6.

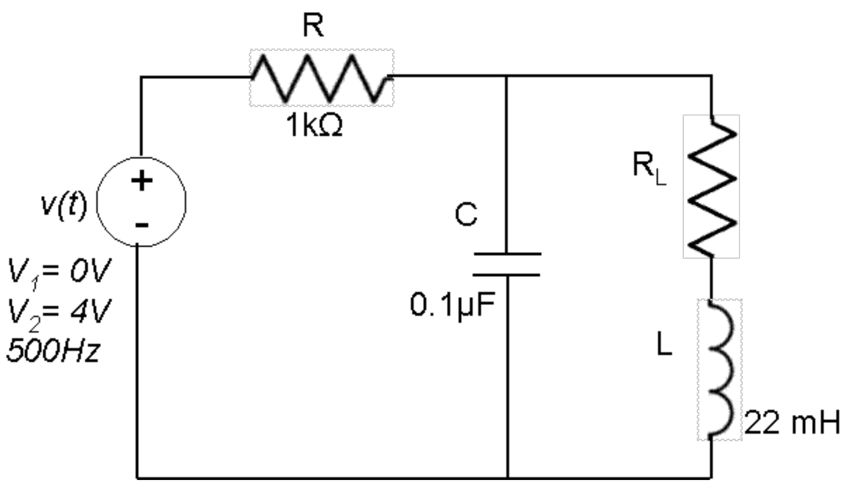


Figure 6 – RLC circuit

9 Give the second order differential equation for the voltage over the capacitor C.
10 Give the solutions for s_1 and s_2.
11 Is this circuit critically, over-, or under-damped?
12 Determine the initial values $v_c(0^-)$, $i_L(0^-)$ and $dv_c(0^+)/dt$ (*Hint:* $dv_c(0^+)/dt$ is nonzero).

13 Determine K_1 and K_2.
14 Determine the final value of the voltage over the capacitor.
15 Determine the period $T_d = 2\pi/\omega_d$ of the oscillation of the voltage across the capacitor.

Method

Now we are going to measure the circuit.
Measure the voltage over the capacitor with the oscilloscope and verify whether the measurements match the calculations done earlier in this experiment.

Execution

Now replace R with a resistor of 100Ω.

16 What response does the circuit have now? Verify your conclusion with a calculation.
17 When is the circuit critically damped? Try to capture this in measurements.

Conclusion

18 Compare theory with practice.

Bonus

You can now put the unknown inductor (from part 1) in the circuit and try to determine its self-inductance and ESR using the theory and measurements of second order circuits.

Experiment 4

The oscilloscope and the probe

Introduction

Problem

As a continuation of the introduction to the oscilloscope and the probe, this lab experiment covers the following topics:

1. analysis of the influence of the oscilloscope on a measuring setup,
2. the use of a measuring probe.

Error analysis

In part I and III, the error analysis is done using the “least squares method” and is explicitly covered. In part II, there is no numerical error analysis.

EXCEL-sheet

For the calculations and the processing of the measured results, a Microsoft-EXCEL sheet has been provided. This sheet allows for partially automatic calculations and can be used to plot graphs as well.

Logging in

For logging in, you will have to use two accounts. First, log in with the code of the practicum. Then continue to log in with your student account.

Part I - The internal impedance of the oscilloscope

Introduction

Just as a voltage meter, an oscilloscope has influence on the measurements, due to its finite internal impedance. In this experiment, we are going to examine the influence of the oscilloscope on a measurement.

Analysis

The load of a measurement setup induced by an oscilloscope

Just like any other measuring instrument, an oscilloscope acts as a load to a circuit with an output impedance of Z_i. This affects the results of the measurements on the system. The circuit in figure 1 illustrates the equivalent of an oscilloscope as a parallel circuit of an internal resistance and an internal capacitance.

1* Give an expression for the internal impedance of the oscilloscope as shown in figure 1.

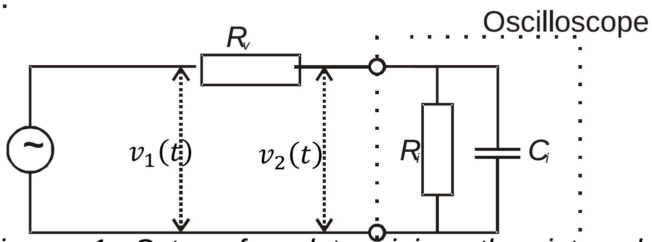


Figure 1 Setup for determining the internal resistance of an oscilloscope

Least Squares Method

The Least Squares Method is commonly used for determining the parameters of a linear relation from a large quantity of measurements. Read GML §4.10 for an explanation on this subject.

2* Briefly summarize the least squares method in your own words.

Method

The internal impedance of the oscilloscope can be determined with the use of figure 1 and a series resistor with resistance R_v. The measured voltages v_1 and v_2 are alternating voltages:

$$v_1(t) = V_{1m}cos(\omega t + \varphi_1) = V_{1m} \cdot Re\{e^{j(\omega t+\varphi_1)}\}$$
$$v_2(t) = V_{2m}cos(\omega t + \varphi_2) = V_{2m} \cdot Re\{e^{j(\omega t+\varphi_2)}\} \quad (1)$$

The corresponding phasors are:

$$\mathrm{V}_1 = V_{1m}e^{j\varphi_1}$$
$$\mathrm{V}_2 = V_{2m}e^{j\varphi_2} \quad (2)$$

In these equations, V_{1m} and V_{2m} are the amplitudes of the AC-signals measured by the oscilloscope. The ratio between these amplitudes can be deduced from the ratio between the corresponding phasors.

3* Calculate the (complex) ratio $\frac{\mathrm{V}_1}{\mathrm{V}_2}$ in terms of the internal impedance of the oscilloscope, the additional resistor R_v and the frequency f of the AC-voltage.

For the relation between the amplitudes, we define:

$$\alpha \equiv \left|\frac{V_{1\mathrm{m}}}{V_{2\mathrm{m}}}\right|^2 \quad (3)$$

We can now determine an expression for $\alpha(f)$, using the result of question 3.
$\alpha(f)$ is in the form of:

$$\alpha(f) = mf^2 + b \quad (4)$$

4* Express m and b in the parameters R_v, R_i and C_i.

5* Calculate R_i and C_i as a function of m, b and R_v.

In summary, we can now determine the internal impedance of the oscilloscope as follows:

- Measure $\alpha(f)$ for several frequencies f and plot these values in a graph.
- Determine m and b.
- Determine the values of R_i and C_i from m, b and R_v.

We now want to plot $\alpha(.)$ in such a way, that it forms a linear relation. In this manner, we can easily check whether equation (4) holds. Furthermore, we can determine m and b using the least squares method.

6* How can we plot $\alpha(.)$ in order to acquire a straight line?

Execution

- Choose your measuring frequencies in such a way, that you get an even distribution over the graph.
- Start EXCEL and open the sheet. Place the values for $\alpha(.)$ in the sheet with the set values for the frequency.
- Determine m and b with the EXCEL function LINEST (Dutch: lijnsch).
- Plot the measurements in one graph with the calculated relation for $\alpha(f)$.
- Determine R_i and C_i and the error margin. This can be done using the EXCEL sheet. For the error margin the GLM §3.8 should be consulted. There some equations are given which should be implemented in EXCEL.

Conclusion

7* Are the calculated values for m and b within the error margin of the measured values?

8* Discuss the observed differences.

Part II - The use of a measuring probe to increase the input impedance of the oscilloscope

Introduction

In this part of the experiment we examine the possibility of increasing the input impedance of an oscilloscope by using a measuring probe between the circuit and the oscilloscope. Herein, of course we would like to keep the shape of the measured signals unchanged, which requires appropriate setting of the probe.

Analysis

A measuring probe can be used to increase the internal impedance of the oscilloscope. Furthermore, it is required that the shape of the signal remains to be unchanged. A description of the input impedance of the oscilloscope and the use of a probe can be read in GLM §4.3.8 and 4.3.9.

9* Show (e.g. using the description in the GLM), that the probe capacitance C_p to be set, can be deduced as:

$$C_p = \frac{R_i C_i}{R_p} \qquad (5)$$

Method

General
Give the measuring setup, using a voltage source, a probe and an oscilloscope (GLM §4.3.9).

Probe compensation
To make the measuring probe purely act as a resistive load, a capacitor is added to the circuit. In our measuring probes, this capacitor can be found near the oscilloscope side of the cord. In this capacitor, there is a small hole, in which you can use a screwdriver to adjust the capacitance. The tuning can most easily be done with the use of a square wave; distortion due to filtering can most easily be noticed using square waves. On the front panel of the oscilloscope, there are two pins especially designed for the tuning of probes: one produces a square wave and the other one is a ground pin. A function generator, however, can also be used.

Method
Fine tune the probe as described above. Give a sketch of the following situations:

- A correctly compensated probe,
- A probe in which the screw has been turned left in case of incorrect setting,
- A probe in which the screw has been turned right in case of incorrect setting.

Execution
We switch the probe attenuation setting to 10x (1:10 mode). Replace the connecting cable between the probe and the oscilloscope by a significant longer or shorter one.

10* Is the image of the square wave on the oscilloscope still undistorted?

11* Determine $\alpha(f)$ again. Does it change? If so, why?

Conclusion
Formulate the conclusion of this lab experiment. In particular discuss the differences observed in part I.

Part III (Bonus) - Determining the total impedance of the oscilloscope with a probe

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Introduction
In this part we are going to check whether the set probe capacitance conform equation (5) is correct. This can be done by determining the total input impedance of the oscilloscope with the probe.

Analysis
We want to determine the total input impedance of an oscilloscope with a probe. This can be done in the same way as how the input impedance of solely the oscilloscope is determined. Next, we want to verify whether the probe capacitance behaves as described in equation (5). Calculate the capacitance C_p of the probe from the input impedance Z_{ik} and the total input impedance of the oscilloscope with probe, Z_p+Z_{ik}.

Method and execution

Perform the steps in part II once again for the combination of the oscilloscope with the probe.

Execution (continued)

Determine C_p from the measurements in part II and part III.

Conclusion

Is the obtained value for C_p in agreement with equation (5)?

Experiment 5

Transfer functions and Bode diagrams of RC and RLC networks

Introduction

Definition of the problem

Study of the transfer functions of RLC and RC circuits (see Fig. 1).
- Determining theoretical Bode diagrams,
- Measurements of the actual Bode diagram.

A harmonic signal at the input of a circuit results in a harmonic signal at the output. The (complex) relationship between these two signals is dependent on the frequency of the input signal. This leads to the concept of the transfer function. A special way of drawing a transfer function is the Bode diagram.
In this experiment, the Bode diagrams of an RLC circuit and an RC circuit will be measured and will be compared with the theory.

LABVIEW and MATLAB

Two Labview (virtual instrumentation) utilities are available for the processing and the calculations on the measuring results: "*Transfer_Function_RLC.vi*" and "*Transfer_Function _RC.vi*". For printing of the graphs, the results can be saved (by default, files will be saved on P:\) and can be read using Matlab scripts "*Transfer_Function _RLC.m*" and "*Transfer_Function _RC.m*". These scripts are used for producing several graphs from which individual graphs can be chosen for printing and journalising. Printing from Labview directly gives a less clear view of the graphs and is therefore not recommended.
Appendix 1 briefly covers the use of Labview and Matlab.

Error analysis

The processing of the measurements on the PC also includes an error analysis. Discuss these results explicitly in your journal and draw conclusions regarding the validity of the theory.

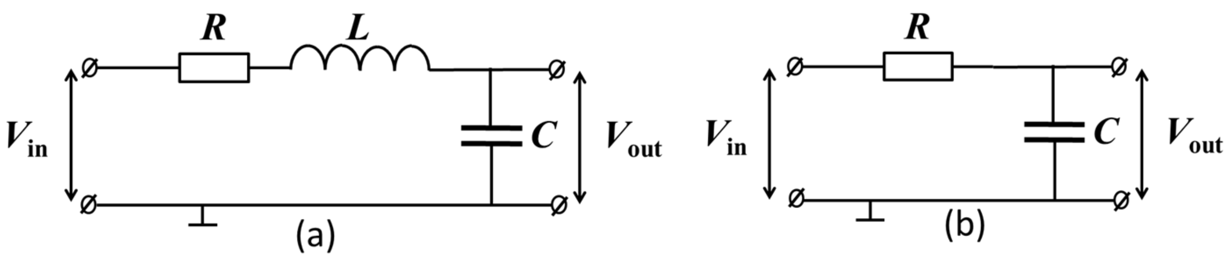


Figure 1: Schematic circuit diagrams of (a) a series RLC network and (b) a series RC network.

Analysis

We consider the following transfer function[1] (Refer to lecture slides)

$$H(j\omega) = \frac{V_{out}(j\omega)}{V_{in}(j\omega)}$$

1 Determine the expression of the transfer function of the series RLC circuit depicted in Fig. 1(a).

2 Express the transfer function as follows:

$$H(j\omega) = \frac{1}{1 + j\frac{\omega}{\omega_0}\frac{1}{Q} + \left(j\frac{\omega}{\omega_0}\right)^2}$$

and determine the parameters ω_0 and Q expressed in terms of R, L and C.

3 Determine the expression of the transfer function of the series RC circuit depicted in Fig. 1(b).

4 For both circuits, specify the expression of the asymptote in the Bode diagrams.

Parameters of an RLC circuit

We are interested in a few parameters that are characteristic of the series RLC circuit:

5 Give an expression for f_0 ($f_0=\omega_0/(2\pi)$), that frequency for which there is a maximum current in the circuit; the circuit's complex impedance will only have a real value at that point.

6 Give the frequency f_{Hmax} for which there is a maximum transfer[2]. Does this correspond to the resonance frequency? Please explain.

Parameters of an RC circuit

The number of parameters of the RC circuit is limited:

7 Give the position of the 3dB point f_{3dB}, for which the transfer is $1/\sqrt{2}$ times the maximum transfer (there is only one point this time).

Precision analysis

Read §3.8 of the GLM about the Statistical estimation of errors.

8 How is the mean squared error defined?

Method

Design a measuring setup and a measuring protocol for the experimental determination of Bode diagrams according the given Labview .vi-files.
Ingredients:

---

[1] In the literature the transfer function is also known as the system function.

[2] This parameter is sometimes also called the resonance frequency, though this is strictly not the case. When discussing a resonance frequency, always specify what you exactly mean.

- Function generator,
- Oscilloscope,
- RLC and RC circuits,
- Measuring bridge for measuring R, L and C.

Measuring points
Pay attention to the choice of measuring points: what regions of the frequency range especially require many measuring points?
Multiple measurements can be done using the Labview .vi-file. It will determine the average Bode diagram and the corresponding mean errors from the measurements. The generated graph in Labview only shows the theoretical curve and the average of the measured curves. The errors will become visible by using the Matlab scripts.

Execution

RLC circuit

1. The theoretical transfer function of the RLC circuit can be calculated using the Labview .vi-file "Transfer_Function_*RLC.vi*". For this to work correctly, the formulas of the amplitude and the phase have to be entered in the formula window. Also give the formula for the calculation of the asymptote.
   Consult the appendix for an explanation of the utility in Labview.

   Beware: π ***=pi(1).***
2. Determine the components' values in the RLC circuit.
   - Fill in the measured values in the corresponding indicator boxes.
3. The Bode diagrams, generated by the Labview .vi-file, can be drawn and printed using the Matlab script "Transfer_Function_RLC.m" (this script can also be used after the measurements are done).
4. Determine the number of measurements that you are going to perform. The Matlab script "Transfer_Function_RLC.m" will derive the mean transfer and the associated standard deviations of these measurements.
5. Measure the transfer function of the RLC circuit using the Labview .vi-file.
6. Draw the average transfer of the measured transfer function along with the theoretical Bode diagram using the Matlab script.
7. Measure the parameters (of the RLC circuit) you calculated earlier in the star questions in the Analysis.
8. Repeat the measurements. Choose the measuring points in such a way that the various parameters can be indicated accurately.

RC circuit

1. The theoretical transfer function of the RC circuit can be calculated using the Labview .vi-file "Transfer_Function_*RC.vi*". For this to work correctly, the formulas of the amplitude and the phase have to be entered in the formula window. Also give the formula for the calculation of the asymptote.
2. Determine the components' values in the RC circuit.
   - Fill in the measured values in the concerning indicator boxes.
3. The Bode diagrams, generated by the Labview .vi-file, can be drawn and printed using the Matlab script "Transfer_Function_RC.m" (this script can also be used after the measurements are done).
4. Determine the number of measurements you are going to perform. The Matlab script "Transfer_Function_RC.m" will derive the mean transfer and the associated standard deviations of these measurements.
5. Measure the transfer function of the RC circuit.

6. Draw the average transfer of the measured transfer function along with the theoretical Bode diagram using the Matlab script.
7. Repeat the measurements. Choose the measuring points in such a way that the frequency, associated with the -3 dB point, can be indicated accurately.

Conclusions

9* Compare the measuring results to the analysis.

10* At which frequency is the transfer function of the RLC circuit at its maximum? Does this correspond to the resonance frequency? Please explain.

11* Compare the measured values of the parameters of the RLC circuit to the theoretical values obtained for star questions *2* and *4*-*6*.

12* Compare the measured values of the parameters of the RC circuit to the theoretical values.

13* What is the relationship between the number of measurements and the standard deviation? How many measurements at least have to be done?

Bonus

14 Give the positions of the 3dB points f_{3dB}, for which the transfer is $1/\sqrt{2}$ times the maximum transfer:

$$H(j\omega_{3dB}) = \frac{1}{\sqrt{2}} H(j\omega_{H\max})$$

15 Give the bandwidth β, the distance between the two 3dB points:

$$\beta = \omega_{1,3dB} - \omega_{2,3dB}$$

16 Give an expression for the quality factor Q.

17 Compare the measured values of the parameters of the RLC circuit to the theoretical values obtained for star questions *14*-*16*.

Experiment 6

Fourier analysis of signals

Introduction

As known from the lectures of the module Electric Circuits, every periodical signal $f(t)$ can be described as the sum of a constant signal and an infinite series of harmonic signals: the *Fourier components* of $f(t)$. The frequencies of these harmonic signals are multiples of the frequency of $f(t)$. A number of expressions for the amplitudes of these harmonic signals has been covered during the lectures. These included one using complex numbers and one using only real numbers. Using one of these functions we can calculate the amplitudes, which are also called the *Fourier coefficients*. Another expression has been covered, which gives the approximation of $f(t)$ when the function is approximated by a couple of its Fourier components.

Definition of the problem

Make sure you have read (and understood) the whole section on this experiment before formulating the definition of the problem. Elements in this definition (the objective) are:

- Fourier analysis (Nilsson chapter 16),
- Square wave signal and triangular wave signal,
- Calculation vs. measurement,
- Examination of the measuring equipment (view the section “Methods” of part 1) and its limitations,
- Compare the practical results with the theory.

Error analysis

No numerical error analysis has to be done during this experiment.

Part I

Analysis

Fourier analysis of a signal

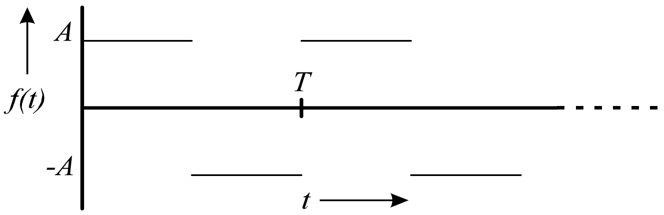


Figure 1 Square wave signal

1 Give (a) general expression(s) for the Fourier coefficients of a signal $f(t)$. Choose either the real or the complex form and explain your choice.

2 Give general expressions which can be used to calculate the Fourier coefficients of the signal $f(t)$ as indicated in figure 1.

3 Using the expression(s) found in 1* and 2*, demonstrate that the (real) Fourier coefficients of the square wave signal, shown in figure 1, meet:

$$a_n = 0 \qquad n \geq 0$$
$$b_n = 0, \qquad n > 0, \qquad n \text{ even}$$
$$b_n = \frac{4A}{n\pi} \quad n > 0, \qquad n \text{ odd}$$

You may also give the complex coefficients.

- You may have already proven this during the seminars. Use this proof!

The bandpass filter

The filter, shown in figure 2, can be used to filter the Fourier components from a signal. The following questions deal with the way this is done.

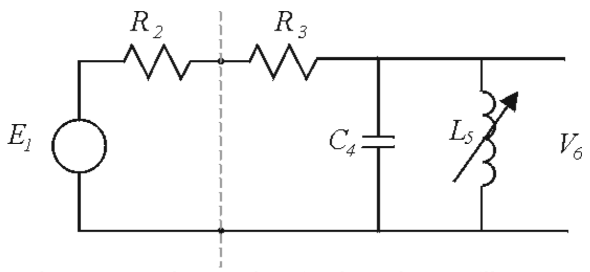


Figure 2 Schematic of a bandpass filter; R_2 is the internal resistance of the supply E_1

4 Demonstrate that the transfer function of the schematic, shown in figure 2, equals:

$$H(j\omega) = \frac{V_6(j\omega)}{E_1(j\omega)} = \frac{j\omega L_5}{j\omega L_5 + (R_2 + R_3)(1 + (j\omega)^2 L_5 C_4)}$$

5 Express ω_0 and Q in the components' values, using the fact that the transfer function can be written as:

$$H(j\omega) = \frac{j\omega/(\omega_0 Q)}{1 + j\omega/(\omega_0 Q) + (j\omega/\omega_0)^2}$$

Hint: The denominators of both expressions for $H(j\omega)$ consist of a constant, a term with ω and a term with ω^2. In the last expression for $H(j\omega)$, there is a 1 in the denominator, so let's make sure that this is also the case in the first expression. So in the first expression, divide the numerator and the denominator by (R_2+R_3). Now the coefficients of ω and ω^2 of both expressions should also be equal.

6 The frequency of the filter is tuned to ensure a maximum transfer for $\omega = \omega_0$ *($f_0 = \omega_0/(2\pi)$)*. Show that the maximum transfer is 1. What is the phase shift of the filter? And what does this mean for the difference in phase between the input and the output signal of the filter when $\omega = \omega_0$?

7 When all filters have been ideally adjusted (when all filters have a difference in phase, between the input and output voltage, of 0˚ when $\omega = \omega_0$ in that filter), what does that say about the output signal of the box, when this is the sum of the output signals of multiple filters?

Methods

When a number of Fourier coefficients of signal *f(t)* is known, an approximation of *f(t)* can be obtained by the sum of the corresponding Fourier components. In this experiment this is done by using the Matlab utility *"Fourier_Analysis_squarewave.m"*. This utility calculates and prints the first, third and fifth harmonic of the square wave as shown in figure 1. The utility first asks for a number of input parameters. When the correct values have been entered, the utility will show four graphs. The first graph shows the three calculated harmonics separately, the second one only shows the first harmonic. The third graph shows the sum of the first and the third harmonic and the last graph shows the sum of all three harmonics.

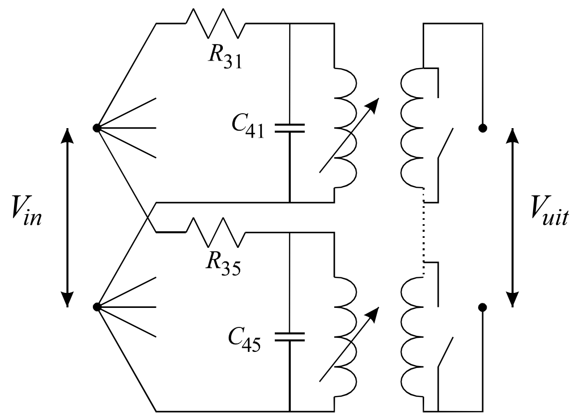


Figure 3 Contents of the box for the measuring of the Fourier components

The Bode diagram of the transfer function of question 4* can be obtained using the Matlab utility "*Fourier_Analysis_filter.m*". When the utility has started, it will ask for a number of input parameters. When these have been entered, the utility will show the amplitude characteristic and the phase characteristic of the filter.

In this experiment, a box is used that generates the first, third and fifth Fourier component of any arbitrary signal of 1 kHz. On the box, there is a switch for each of the components. A combination of switches will result in an output voltage that is the sum of the corresponding Fourier components. This signal can be viewed using the oscilloscope.

The box contains four filters. Figure 2 shows the structure of a filter. Each of these filters has been tuned to the frequency of the Fourier component that has to be available at the filter's output. Figure 3 shows how it is achieved that the sum of a certain combination of Fourier components is available at the output. It can be seen that each of the inductors of the filters is used as the primary coil of a transformer. The secondary coils of the four transformers are connected in series. In parallel to each secondary coil is a switch. When a switch is turned to "0", its corresponding coil is shorted. By turning one or more switches to "+", the shorts change to open connections and the sum of the voltages on the coils will be available on the output (they are connected in series!).

Beware! The transformers in the Fourier boxes have a transfer ratio of 10:1. So $V_{out}=1/10 \cdot V_{in}$. Take this into account!

Execution

1. On the PC, calculate the first, third and fifth harmonic of the square wave signal using the utility "*Fourier_Analysis_squarewave.m*" and print them separately. Print the sum of a few combinations of these components as well. Include these graphs in your journal.
2. Plot the Bode diagram of the filter in figure 2 using the Matlab utility "*Fourier_Analysis_filter.m*". Use the following element values:
   - R_2 = 50 Ω (internal resistance of the function generator!),
   - R_3 = 20 kΩ,
   - C_4 = 100 nF,
   - L_5 = 237 mH.

These are the values for the components for the filter that is tuned to 1 kHz. Add the result to your journal.

8	What are the values of ω_0 and Q in this filter? How does this reflect on the Bode diagram of the filter?

For the sake of completeness, it is mentioned that the values of the resistors and the capacitors in the other filters are equal to the values mentioned above. The values of L_5, however, are 58.1mH, 25.6mH and 9.96mH in the filters tuned to 2, 3 and 5 kHz respectively.

3. Also sketch the amplitude characteristics of the filter that is tuned to 1 kHz by doing several measurements in the frequency range that is of interest. The printing of the obtained graph can be done using the Labview file "*Fourier_Analysis_transfer.vi*" in combination with the Matlab file "*Fourier_Analysis_transfer.m*".
4. Now use the provided box to examine the Fourier components of a 1 kHz, square wave signal produced by the function generator. Draw the used circuit. Investigate the output signal at various positions of the switches. Take meaningful observations, like the measured first harmonic, the sum of the first and the third harmonic and the sum of the first, third and fifth harmonic, and add them to the journal. What does the measured second harmonic look like? Explain the measured signal.
   The Labview vi-file "*Fourier_Analysis_measurement.vi*" can be used when doing measurements. In this file, the shape (triangle or square) and the frequency of the signal can be entered. The input and output signals of the filter will be saved in a file that can be read using the Matlab script "*Fourier_Analysis_measurement.m*".

Part II

In part 2, a triangular wave signal, as depicted in figure 4, will be examined in a similar way as the square wave in part 1.

Analysis

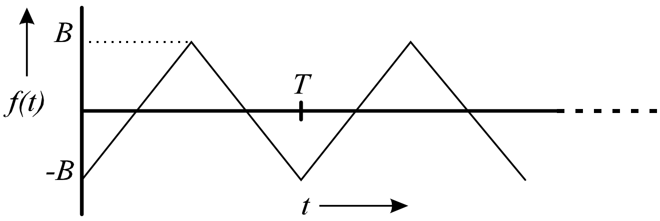


Figure 4 Triangular wave signal

9	Using the expression(s) found in *1* and *2*, demonstrate that the Fourier coefficients of the triangular wave signal, shown in figure 4, meet:

$$
\begin{aligned}
a_0 &= 0 & & \\
a_n &= 0 & n > 0, \quad & n \text{ even} \\
a_n &= \frac{-8B}{(n\pi)^2} & n > 0, \quad & n \text{ odd} \\
b_n &= 0 & n > 0 &
\end{aligned}
$$

(This question can be answered in two different ways: on the one hand by calculating the integral as in *3* (which is quite some work), on the other hand by realising that the triangular wave signal in figure 4 is in fact an integrated square wave signal. If this signal is the integrated square wave signal of figure 2, what is B expressed in A, and what are the effects for the Fourier components of the triangular wave signal?)

Like in part 1, you may have already done this calculation during the seminars.

Methods/execution

Plot the Fourier components of the triangular wave signal of figure 4 in the same way as in part 1, using the Matlab file “*Fourier_Analysis_triangle.m*”. Also measure these components using the oscilloscope and “*Fourier_Analysis_measurement.vi*”. Add all meaningful results to your journal.

Conclusions

10* To what extent do the calculated and the measured results agree?

11* Are there any recognisable differences between the results of the examination of the square wave signal and those of the triangular wave signal?

12* What are the possible causes for any potential deviations between the calculated and the measured results? (Think of the function generator and ask yourself what kind of influence a small error in the filter’s adjustment can have regarding the difference in phase between the input and the output of a filter.)

Experiment 7

The black-box model

Introduction

Figure 1: Photographs of the black-box container seen from the top (left) and from the side (right). Geert Jan Laanstra, with courtesy.

The main objective of this experiment is to study the use of convolution in practice. For this we provide a black-box container in which a certain circuit has been incorporated (see Fig. 1). This container (which is actually not black but grey) however should not be opened and should only be electrically measured. The final objective of this experiment is to determine the content, hence the electrical circuit, of this black-box container. But before we do so, we would like to perform the analysis.

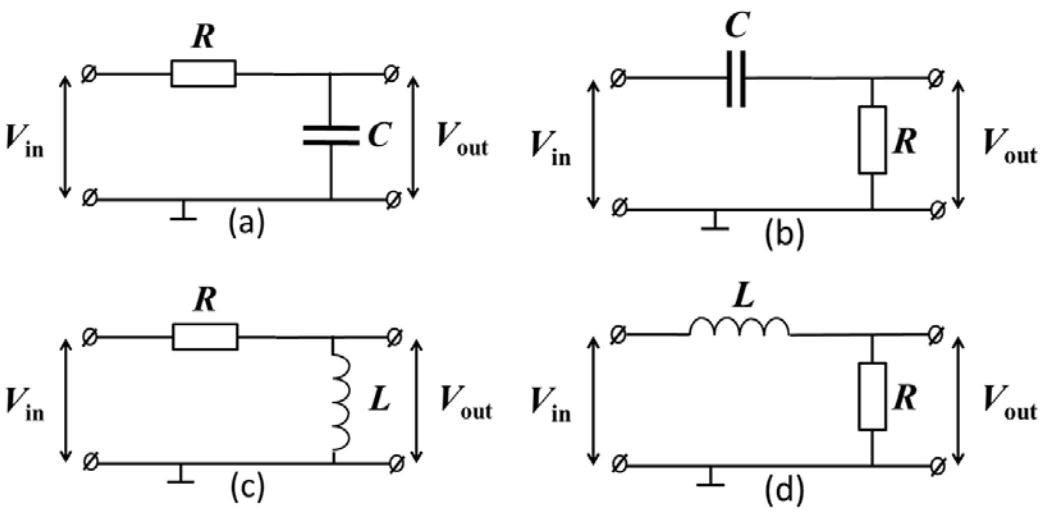


Figure 2: Different first order circuits: (a), (b) an RC circuit and (c), (d) an RL circuit.

Analysis

Consider the following first order circuits depicted in Fig. 2.

1 For each of these circuits find the differential equation (DE) expressed in V_{out}(t).

2 Solve the DE for the unit step function $u(t)$ as input.

Next we will study the effect of a unit impulse function which is used as an input signal to the circuit.

Basically we can say that the output signal in time domain can be described as follows:

$$y(t) = \int_{\tau=-\infty}^{\infty} x(\tau) \cdot h(t-\tau) d\tau$$

where $x(t)$ is the input signal $(=V_{in}(t))$. Note that $y(t)=V_{out}(t)$.

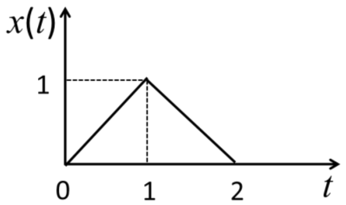
3 Now let's assume that $x(t) = \delta(t)$, *i.e.* a unit impulse function. What are the unique properties of this function?

4 Now determine the impulse response $h(t)$ by solving the DE for $x(t) = \delta(t)$.

5 Now let's assume that $x(t) = u(t) - u(t - \Delta t)$, where $u(t)$ *is* a unit step function. Give the mathematical expressions for $y(t)$ for the systems shown in Figs. 2(a) and 2(c).

6 We make life more complicated and come up with a more complex input signal. Let's assume the following:

$$x(t) = \begin{cases} 0 & t < 0 \\ t & 0 \le t < 1 \\ 2-t & 1 \le t < 2 \\ 0 & t \ge 2 \end{cases}$$



Now determine $y(t)$ for the systems shown in Figs. 1(a) and 1(c).

Method

Now we would like to perform the analysis described above in practice. For this we provide a black-box container in which a certain circuit has been incorporated which should not be opened.

The approach would be to determine the transfer function of the electric circuit inside the container first. Then some additional experiments will be performed to verify this. Finally, the internal electric circuit in the black-box is determined.

7 Please make a sketch of the schematic measurement circuit to study the convolution principle and explain. Also write down what possible internal parasitic elements could be present from the power supply and measurement tools, so please check the output impedance of the function generator and the input impedance of the oscilloscope.

8 Discuss all individual elements of your measurement circuit and how they affect the measurements.

Execution

Build your proposed measurement setup accordingly.

Ingredients:
- Function generator,
- Oscilloscope,
- Black-box container,
- Cabling.

Do not forget to state the settings of the measurement equipment in view of the reproducibility requirement!!!

Now we would like to determine the transfer function of the unknown circuit.

9 According to theory what input signal(s) should be applied for determining the transfer function?

10 How does the output signal look like after applying the intended input signal of question 9? A picture tells you more than a thousand words…

11 Determine the time constant τ of the circuit.

12 Determine the transfer function of the circuit.

13 Now apply a triangular input wave to the circuit. Do you obtain the expected output signal? Please explain.

Conclusion

14 What circuit do you expect to be inside the black-box? Are there any doubts related to possible errors which could affect your conclusion? If so, in what way are the results affected?

Experiment 8

The transformer as two-port

Introduction

During the lectures a two-port is defined as a black-box with four terminals, two as input, and two as output. Over the terminals on the input side is a voltage and through the same terminals a current flows; the output terminals are defined in the same way. The relation between these four physical quantities is fixed, provided that the two-port parameters are known. As stated in chapter 18 of Nilsson, we do not have to know the internal architecture of the two-port to determine its outside behaviour. Agreed, but before we take this for granted, we are going to test this first. We first determine and check the Z-parameters of a transformer. After that we are going to take a look at the behaviour of two transformers in series. Analysing this circuit using the two-port method is rather easy, which gives an impression on the power of this method.

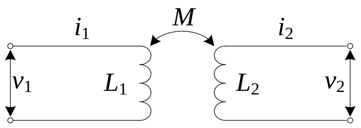
Part I - Z-parameters of a two-port

Goal of the experiment

We are going to determine the Z-parameters of a transformer.

Analysis

We often use the results of the hand-out two-ports, which was provided during the lectures.
A transformer comprises two inductively coupled inductors.



We want to describe a transformer using Z-parameters. (See hand-out)

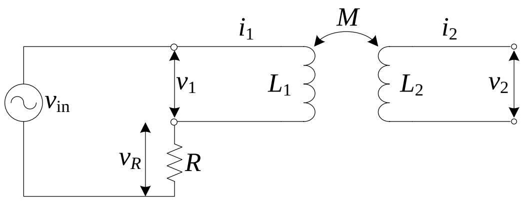
1 Give the Voltage-Current equations for a two-port with Z-parameters.

2 Give an expression of the Z-parameters as a function of the induction-coefficients L_1, L_2 and the mutual inductance M.

3 Give an expression for the coupling coefficient k of the transformer.

Method

The two-port parameters are measured using the following measurement setup



Resistor R has two functions:

1. We use it as a measurement-resistor, which enables us to measure i_1,
2. We use it as a current limiter which protects the transformer at low frequencies (v_{in}).

We use the phase of the voltage v_1 as a reference, which we define to be zero.

This results in:

$$\begin{aligned} v_1(t) &= V_{01}\cos(\omega t) \\ i_1(t) &= \frac{v_R(t)}{R} = \frac{V_{0R}\cos(\omega t+\varphi_1)}{R} \\ v_2(t) &= V_{02}\cos(\omega t+\varphi_2) \end{aligned} \tag{6}$$

with φ_1 being the phase-angle between the voltage and the current at the input side of the transformer and φ_2 the phase-angle between the output- and input-voltage.
The measurements are: $\{V_{01} \quad V_{0R} \quad \varphi_1 \quad V_{02} \quad \varphi_2\}$.

4 Prove that z_{11} and z_{12} can be calculated according to:

$$z_{11} = \frac{RV_{01}}{V_{0R}}\exp\left(-j\varphi_1\right) \quad z_{21} = \frac{RV_{02}}{V_{0R}}\exp\left\{j\left(\varphi_2-\varphi_1\right)\right\} \tag{7}$$

The component values of the transformer can be calculated using the equations you found in *2*.

Consequently what values should φ_1 and φ_2 have?

5 Give expressions for z_{11} and z_{21} as a function of the measured phase-angles.
Note!!! The phase-relations only hold when the secondary side of the transformer has no load. Under normal use the secondary side of the transformer has a load, which results in different phase-relations. The two other z-parameters, z_{12} and z_{22} can be found if the experiment is reversed.

Measurement setup

When looking at this measurement setup we have some issues:

- We want to measure three voltages at the same time: v_R, v_1 en v_2. We only have 3 measurement instruments, the oscilloscope with 2 channels, and two multimeters.
- The function generator and both channels of the oscilloscope are grounded. The grounds in this circuit must be at the same node: in a circuit there should only be one ground.

6 Could it be that in a circuit with a transformer there may be two ground points?
A suggestion: measure v_R with the oscilloscope. Measure the other two voltages with multimeters.
7 What is the relation between the peak-to-peak voltage (measured by the oscilloscope) and the RMS voltage which is measured with a multimeter?

Execution

An excel sheet is available for the processing of the measurements Proef7.xls

I. Use the inductance with 300 windings as primary side

Choose $R \approx 3000\Omega$ and $v_i = 5V$ (peak-peak). Check the phase-relations you just calculated with the oscilloscope. Then sweep the frequencies between 100 and 600Hz in 100Hz steps and measure V_{0R}, v_1 and v_2. Make a graph with z_{11} and z_{21} as function of frequency. Determine L_1 and M with the least squares method.

II. Use the inductance with 1500 windings as primary side

Choose $R \approx 20k\Omega$ and $v_{in} = 5V$ (peak-peak). Sweep the frequencies between 100 and 600Hz in 100Hz steps. Check for every frequency the phase-relation and measure the amplitudes. Make a graph with z_{22} and z_{12} as function of frequency. Determine L_1 and M with the least squares method.

Calculate the coupling coefficient k of the transformer

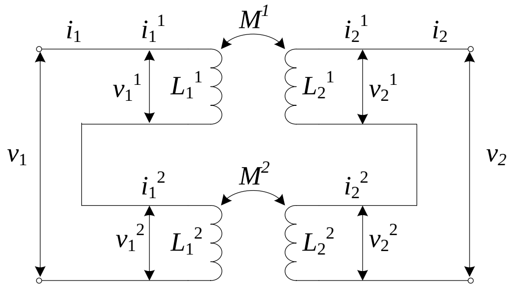
Conclusion

Are the curves straight? Do the values of L_1 and M match your expectations?
Is the transformer ideal?

Part II - Two cascaded transformers

Analysis

The figure below shows a circuit with two transformers in "series".



The behaviour of both transformers can be described separately by the z-parameters found in part 1. This gives us:

$$\begin{aligned} V_1^1 &= z_{11}^1 I_1^1 + z_{12}^1 I_2^1 \qquad & V_1^2 &= z_{11}^2 I_1^2 + z_{12}^2 I_2^2 \\ V_2^1 &= z_{21}^1 I_1^1 + z_{22}^1 I_2^1 \qquad & V_2^2 &= z_{21}^2 I_1^2 + z_{22}^2 I_2^2 \end{aligned} \tag{8}$$

in which the superscript ^{1} indicates transformer 1 and superscript ^{2} indicates transformer 2
The circuit as a whole is described by the original equations:

$$\begin{aligned} V_1 &= z_{11} I_1 + z_{12} I_2 \\ V_2 &= z_{21} I_1 + z_{22} I_2 \end{aligned} \tag{9}$$

Now we want to describe the whole circuit in terms of the z-parameters of the single transformers.

8* Prove the following:

$$z_{11} = z_{11}^1 + z_{11}^2 \quad z_{12} = z_{12}^1 + z_{12}^2$$
$$z_{21} = z_{21}^1 + z_{21}^2 \quad z_{22} = z_{22}^1 + z_{22}^2 \qquad (10)$$

I.e. two transformers placed in series, results in a transformer with its self-inductance and mutual inductance being the sum of those of the individual transformers.

Method

The z-parameters of the combination transformer can be determined in the same way as part 1 with the single transformer.

Execution

The amount of available transformers is limited. Moreover, the results of the measurements can only be checked after both transforms are analysed on their own.
Therefore, this part is executed by groups of four people (two groups together). Every group of four students has then 2 transformers at their disposal.

I. Use both transformers with the 300 windings as primary side

Choose $R \approx 6000\Omega$
Measure parameters z_{11} and z_{21} of the combined transformer at 400Hz (because of time limitations, you do not have to use the least squares method)

- Calculate L_1 and M.

II. Use both transformers with the 1500 windings as primary side

Choose $R \approx 40k\Omega$
Measure parameters z_{22} en z_{12} of the combined transformer at 400Hz (because of time limitations, you do not have to use the least squares method)
Calculate L_2, M and the coupling coefficient k.

- Check whether relation (10) is correct.

Conclusion

Are the results in accordance with (10)?

Appendix I

Use of Labview and Matlab

Labview .vi-files

In some experiments in this lab course, the program Labview is used. With the aid of this programme, a measuring setup can be extended with 'virtual instrumentation' to automatically conduct measurements and process the data of these measurements. For example in lab 4, a function generator is controlled via the .vi-file "Transfer_Function_*RLC.vi*". The raw measurement data is collected automatically from the oscilloscope as well. The construction of the measurement setup remains the same as when manually conducting these experiments; the function generator and oscilloscope have already been connected to the computer. After initializing the programme with the .vi-files, the experiment proceeds as shown on the display. For the actual running of the .vi-file, click the white arrow in the upper left corner. As long as the programme is running, this arrow remains black. The programme can be stopped with the 'cancel' or 'stop' buttons in the programme, or using the red, circular button on top of the screen.
With the tab key, it is possible to change the function of the cursor. This, for example, is needed to type in textboxes. The 'hand' can be used to increase or decrease values using the up- and down keys

Formulas in Labview

In lab 4, there is an assignment to enter a formula, needed to test the measuring results. Apart from the standard mathematical operations (+, -, *, /, etc.), the next functions, amongst others, can be used to formulate formulas in Labview.

abs(x)	Absolute value of x		
acos(x)	Inverse cosine of x in radians		
asin(x)	Inverse sine of x in radians		
atan(x)	Inverse tangent of x in radians		
cos(x)	cosine of x in radians		
exp(x)	exponential power (e^x)		
ln(x)	Natural logarithm of x		
log(x)	Logarithm of x (base 10)		
log2(x)	Logarithm of x (base 2).		
pi(x)	pi=3.14159...	pi(x)=x*p	
	pi(1)=pi		
	pi(2.4)=2.4*p		
sin(x)	Sine of x in radians		
sqrt(x)	Square root of x		
step(x)	Step function	step(x)=0 for x < 0	step(x)=1 for x ≥ 0
tan(x)	Tangent of x in radians		
x^y	x to the power of y		

The parameters for the defining of variables and constants used in the programme have already been defined. These are displayed in the window (please do not try to add your own variables, as these will not be recognised).

Matlab m-files

In combination with Labview we use Matlab. Matlab is more capable of producing graphs, which is why we use Matlab for plotting and printing of the acquired graphs from Labview. The files can be saved in Labview by typing a filename where asked. These files will be automatically saved to “P:\” (each account has an unique P:\ disk).

Matlab files will be placed on Blackboard.

When Matlab has started, make sure to enter “N:\Opgaven\Mina” as ‘current directory’. The name of the script to be executed can be entered in the Command Window (e.g. *experiment4_RLC*). The rest continues automatically.

Appendix II

Self-inductions datasheets

In exercise 3 a number of self-inductions will be used, in this appendix you will find their datasheets.